Compound Inequalities

Work within a small group to answer these questions. Do not race through the exercises on your own. Always make sure that your entire group feels good about a question and answer before you move to the next exercise. Ask your group mates for explanations if you feel uncertain about something, and offer your explanations to others when you understand an exercise but someone else may not.

1. Solve each compound inequality.

use your graphing technology for all such parts

a) $x-1<5$ or $2x>30$
Solve algebraically *and* graphically.

$\Rightarrow x<4$ or $x>15$

Solution set is $(-\infty,4)\cup(15,\infty)$

b) $3x<-5$ or $2x\geq 3$
Solve algebraically.

$\Rightarrow x<\frac{-5}{3}$ or $x\geq\frac{3}{2}$

Solution set is $(-\infty,\frac{-5}{3})\cup[\frac{3}{2},\infty)$

c) $-27\leq 3x\leq 9$
Solve algebraically *and* graphically.

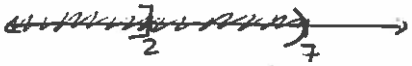
$\Rightarrow -9\leq x\leq 3$

Solution set is $[-9,3]$

d) $2-x>-5$ or $2+x\leq 4$
Solve algebraically *and* graphically.

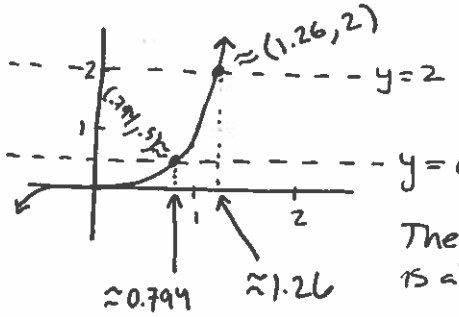
$\Rightarrow -x>-7$ or $x\leq 2$

$\Rightarrow x<7$ or $x\leq 2$



Solution set is $(-\infty,7)$

e) $0.5<x^3\leq 2$
Solve graphically.



The solution set is about $(0.794, 1.26]$.

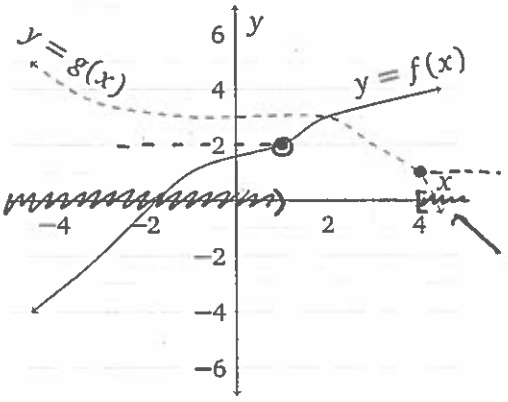
f) $-2\leq 5-\frac{1}{3}x<2$
Solve algebraically *and* graphically.

$\Rightarrow -7\leq -\frac{1}{3}x<-3$

$\Rightarrow 21\geq x>9$

The solution set is $(9,21]$.

2. Here are the graphs of f (solid) and g (dashed). Solve the compound inequality $f(x) < 2$ or $g(x) \leq 1$.

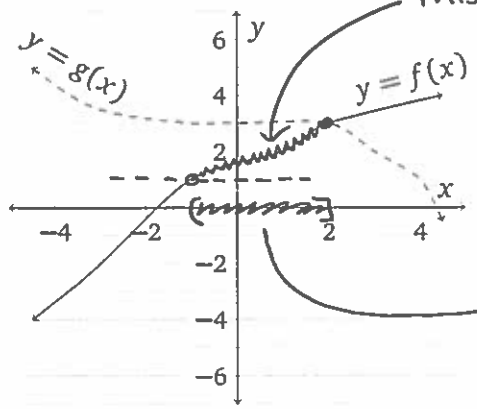


For these x, $f(x) < 2$

For these x, $g(x) \leq 1$

$(-\infty, 1) \cup [4, \infty)$

3. Here are the graphs of f (solid) and g (dashed). Solve the compound inequality $1 < f(x) < g(x)$.



This portion of f's curve is between $y=1$ and $y=g(x)$.

$(-1, 2]$

4. In the following problems, make sure that you set up this problem with a inequalities rather than equations (and that you see why that is appropriate for this problem). Use interval notation. Write a conclusion sentence with context.

(a) If the dew point D on the ground is 60°F, then the dew point for a community x miles above sea level is given by $D(x) = 60 - 5.8x$. Find the altitudes where the dew point is between 57.1°F and 51.3°F.

$57.1 \geq 60 - 5.8x \geq 51.3$

$-2.9 \geq -5.8x \geq -8.7$

$\frac{-2.9}{-5.8} \leq x \leq \frac{-8.7}{-5.8} \implies \frac{1}{2} \leq x \leq \frac{3}{2}$

So the dew pt is between 57.1°F and 51.3°F at altitudes between $\frac{1}{2}$ and $\frac{3}{2}$ miles above sea level.

(b) The formula $F = \frac{9}{5}C + 32$ may be used to convert Celsius temperatures to Fahrenheit temperatures. The greatest temperature ranges on Earth for one place are recorded in Siberia where the temperature has varied from −90°F to 98°F. Set up (and then solve) a compound inequality that describes Fahrenheit temperatures that can happen in Siberia, but whose solutions describe Celsius temperatures that can happen in Siberia.

$-90 \leq \frac{9}{5}C + 32 \leq 98$

$-122 \leq \frac{9}{5}C \leq 66$

$-122 \cdot \frac{5}{9} \leq C \leq 66 \cdot \frac{5}{9}$

$-67.\overline{7} \leq C \leq 36.\overline{6}$

So in Siberia the temperature ranges between $-67.\overline{7}$°C and 36.6°C.