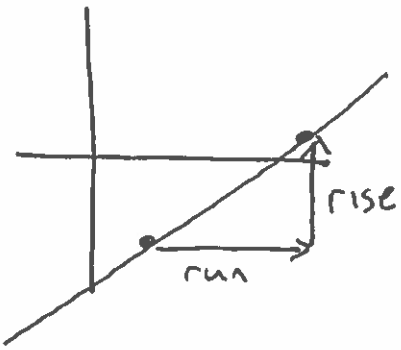
3.3 Slope

$$m = \frac{\text{rise}}{\text{run}} \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

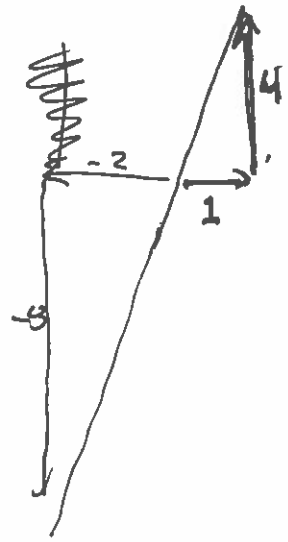


x_1 y_1 x_2 y_2

Ex Line between $(1, 3)$, and $(-1, -5)$, find its slope.

1st point 2nd pt

$$m = \frac{-5 - 3}{-1 - 1} = \frac{-8}{-2} = 4$$



Find equation of this line in slope-intercept form

$$y = mx + b$$

$$y = 4x + b$$ ← true for all (x, y) on the line.

$3 = 4(1) + b$ ← used one point.

$3 = 4 + b$

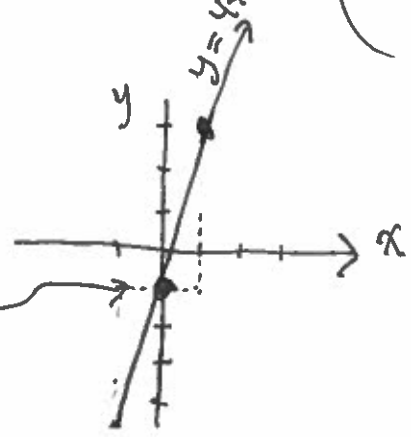
$3 - 4 = 4 + b - 4$

$-1 = b$

$y = 4x + (-1)$

$y = 4x - 1$

Plot graph:



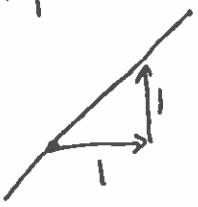
$-1 \rightsquigarrow (0, -1)$
y-intercept

Ex Line through $(1,1)$ and $(-2,-2)$.

- slope?
- equation in slope-intercept form?
- graph?

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-1}{-2-1} = \frac{-3}{-3} = 1$$

slope



$y = m \cdot x + b$

$y = 1 \cdot x + b$

$y = x + b$

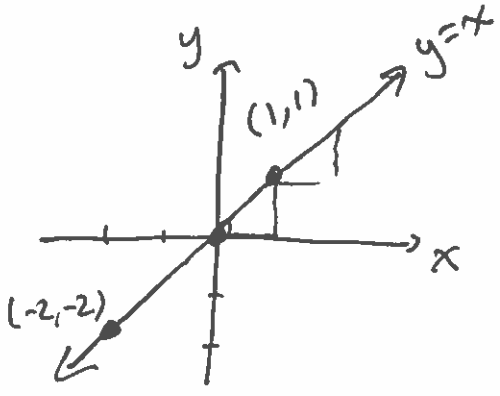
at $(1,1)$

$1 = 1 + b$

$b = 0$

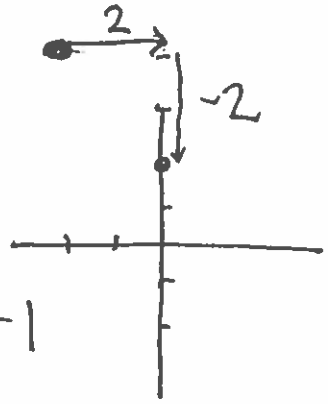
$y = x + 0$

$y = x$



→ is on y-axis ⟹ $b = 2$

Ex Given $(0,2)$ and $(-2,4)$, what's the slope between these points?



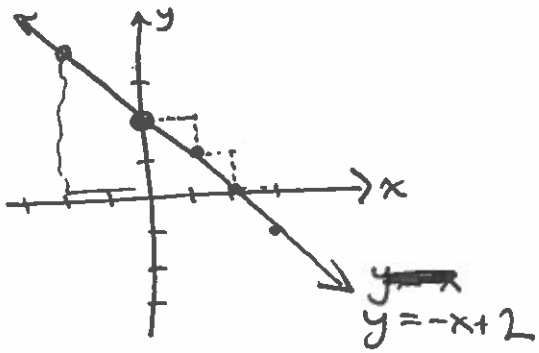
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\text{rise}}{\text{run}} = \frac{-2}{2} = -1$$

Slope-intercept form

$$y = mx + b$$
$$y = -1 \cdot x + b$$
$$y = -x + b$$
$$y = -x + 2$$

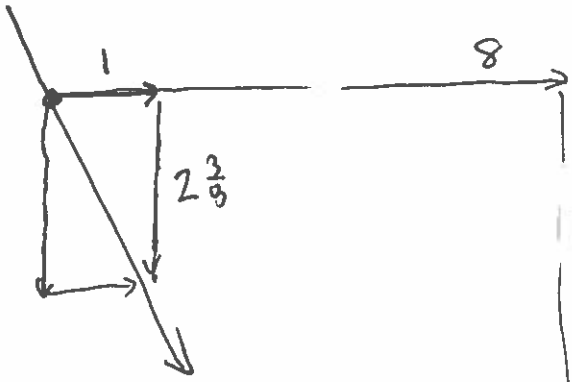
Sketch:



(Check original points...)

Ex A line passe through $(5,17)$ and $(13,-2)$.

Find slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 17}{13 - 5} = \frac{-19}{8}$



$$= \frac{-2\frac{3}{8}}{1}$$

Find slope-intercept equation...

$y = mx + b$

$y = \frac{-19}{8}x + b$

one y → 17, one x → 5

What # is b?

$17 = \frac{-19}{8}(5) + b$

$8 \cdot 17 = 8\left(\frac{-19}{8}(5) + b\right)$

$136 = -19(5) + 8b$

$136 = -95 + 8b$

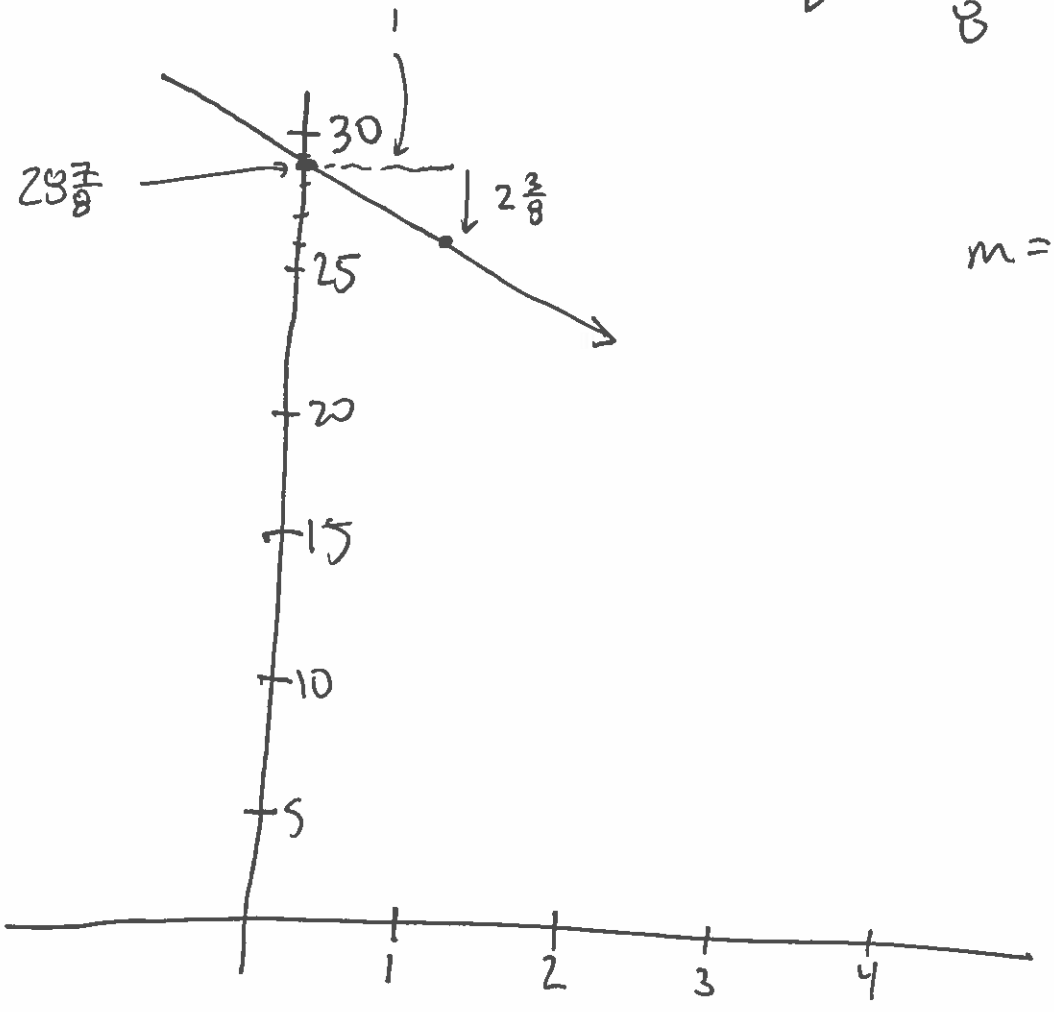
$136 + 95 = -95 + 8b + 95$

$231 = 8b$

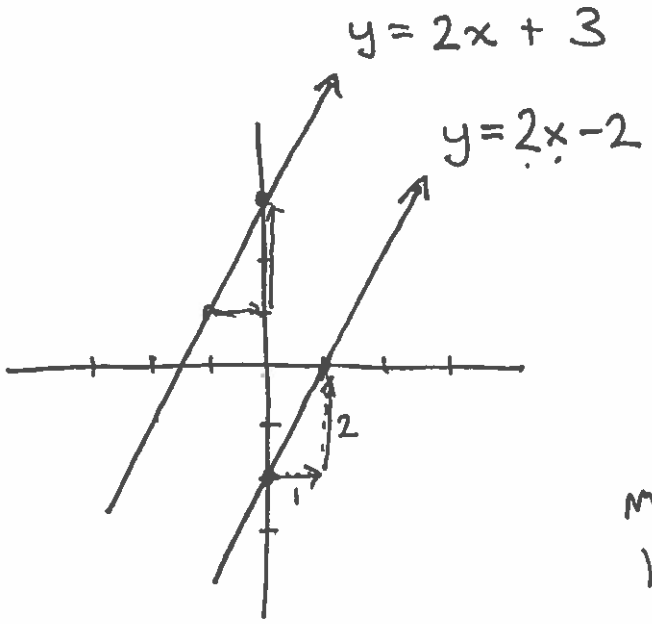
$b = \frac{231}{8}$

$28\frac{7}{8}$

$y = \frac{-19}{8}x + \frac{231}{8}$



$m = \frac{-19}{8} = -2\frac{3}{8}$



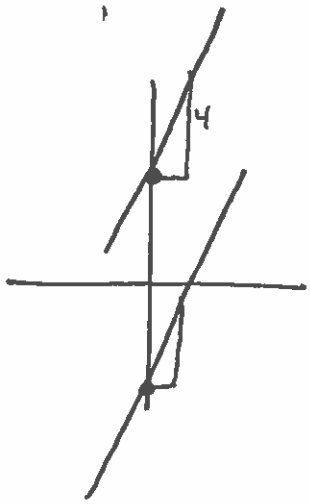
parallel : equally-spaced apart, never cross

means the two lines have same slope.

Ex $y = 4x + 5$

$y = 4x - 5$

slope is 4 for both, so parallel!



Ex A line passes through (2,3) and (5,5).

$$m = \frac{5-3}{5-2} = \frac{2}{3}$$

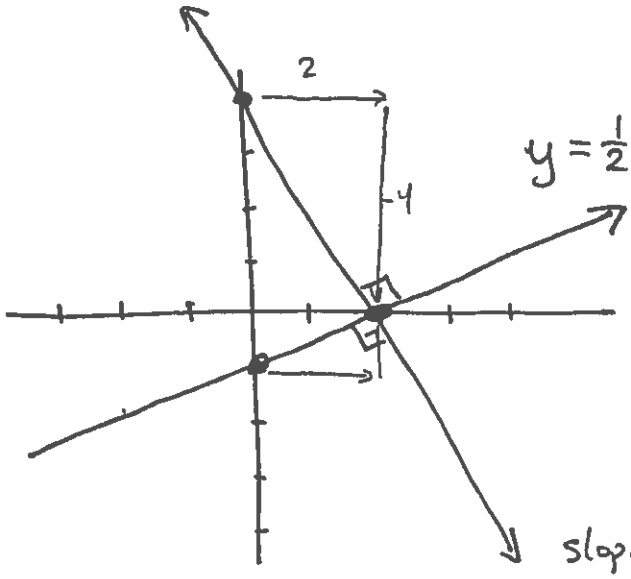
Another line passes through (-2,8) and (1,12).

$$m = \frac{12-8}{1-(-2)} = \frac{4}{3}$$

Are they parallel?

different slopes! $\frac{2}{3} \neq \frac{4}{3}$ $\Longrightarrow$ they are not parallel!

Ex



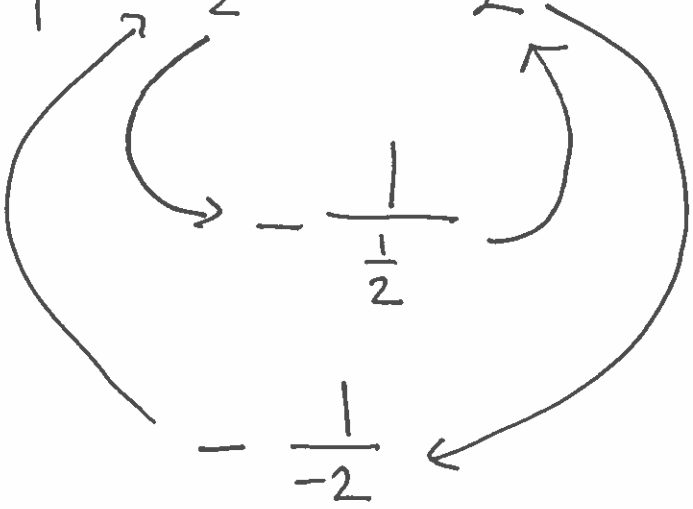
$y = \frac{1}{2}x - 1$

perpendicular: intersects at 90° angle

slope $= \frac{-4}{2} = -2$

$y = -2x + 4$

two slopes: $\frac{1}{2}$ and -2 (not equal $\Rightarrow$ not parallel)



Fact: When lines are perpendicular, slopes are negative reciprocals of each other

$\frac{1}{\text{whatever}}$

Ex $y = \frac{2}{5}x + 10$

$y = -\frac{5}{2}x - 3$

⟹ these are perpendicular

$\frac{2}{5}$ slope & $-\frac{5}{2}$ slope

negative reciprocals!

Ex A line passes through $(0,-1)$ and $(1,2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-1)}{1-0} = \frac{3}{1} = 3$$

Another line passes through $(-4,1)$ and $(4,-1)$.

Are these lines parallel, perpendicular, or neither?

$$m = \frac{-1-1}{4-(-4)} = \frac{-2}{8} = -\frac{1}{4}$$

slopes: $3, -\frac{1}{4}$

not equal ⟹ not parallel

not neg. reciprocals ⟹ not perpendic.

Ex $-9x + y = 5$ Solve for y.

$-9x + y + 9x = 5 + 9x$

$y = 5 + 9x$ ← slope-intercept form

$y = 9x + 5$

slope: 9 y-intercept: 5 (0, 5)

Generic Linear Equation

Can figure out slope and y-intercept...

Ex $8x + y = 0$

Find slope & y-intercept.

Want: $y = mx + b$ (solve for x)

$8x + y - 8x = 0 - 8x$

$y = -8x$

slope -8

y-intercept: $(0, 0)$

Ex $5x - 2y = 10$ What is its slope? y-intercept?

$5x - 2y - 5x = 10 - 5x$

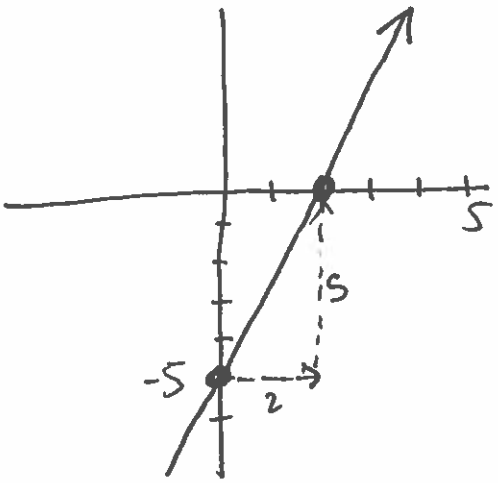
$-2y = -5x + 10$

$\frac{-2y}{-2} = \frac{-5x + 10}{-2}$ Distribute division.

$y = \frac{-5x}{-2} + \frac{10}{-2} = \left(\frac{5}{2}\right)x - 5$

$(0, -5)$

Graph $5x-2y=10$



y-intercept: $(0,-5)$

slope $= \frac{5}{2} \leftarrow$ rise / run

Ex $2x - 3y + 8 = x + 5y + 7$

Has a linear graph (no powers of $x, y, \ldots$)

want to rearrange into $y=mx+b$ form. want to solve for y.

Has a slope.

Has a y-intercept

$2x - 3y + 8 - 2x = x + 5y + 7 - 2x$

$-3y + 8 = -x + 5y + 7$

$-3y + 8 - 8 = -x + 5y + 7 - 8$

$-3y = -x + 5y - 1$

$-3y - 5y = -x + 5y - 1 - 5y$

$-8y = -x - 1$

$\frac{-8y}{-8} = \frac{-x-1}{-8}$

$y = \frac{-x}{-8} - \frac{1}{-8} = \frac{1}{8}x + \frac{1}{8}$

slope $= \frac{1}{8}$

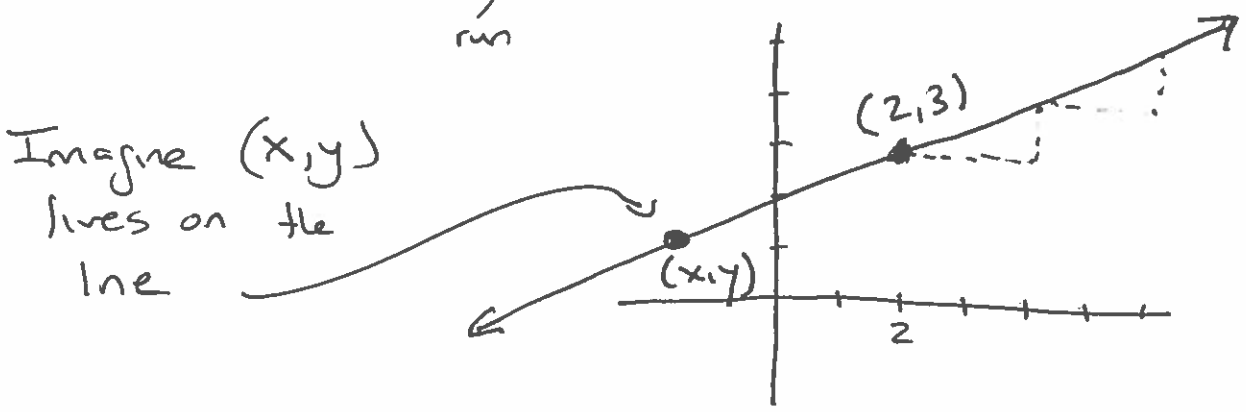
y-intercept $(0, \frac{1}{8})$

3.5 Point-Slope form

Slope-intercept form
$y = mx + b$

Consider a line through $(2, 3)$ with slope $\frac{1}{2}$ ← rise, run

visualize



on the one hand slope is $\frac{1}{2}$

on the other hand $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{y-3}{x-2}$

So $\frac{1}{2} = \frac{y-3}{x-2}$

$\Rightarrow \frac{1}{2}(x-2) = \frac{y-3}{x-2}(x-2)$

$\Rightarrow y - 3 = \frac{1}{2}(x-2)$

this equation is a line equation...

$(2,3)$ original point

slope

$$y - y_1 = m(x - x_1)$$

Point-slope form

y_1: special y-value

m: slope

x_1: special x-value

(x_1, y_1) IS a point on the line!

Ex There's a line with slope 3.
It passes through $(2, 4)$.

Write an equation for this line.

$$y - 4 = 3(x - 2)$$ Done!

Write the line in slope-intercept form ($y = mx + b$)

$$y - 4 = 3x - 6$$
$$y = 3x - 6 + 4$$
$$y = 3x - 2$$

Ex A line has slope -2, passes through $(3,-5)$.

- Write an equation for this line:

$$y-(-5) = -2(x-3)$$
$$y+5 = -2(x-3)$$

- Write slope-intercept equation for this line.

$$y+5 = -2x+6$$
$$y = -2x+1$$

Ex A line passes through $(2,0)$ and $(4,2)$.

Find an equation for this line.

Go for point-slope form...

✓ ? $\quad m = \frac{2-0}{4-2} = \frac{2}{2} = 1$

$$y-y_1 = m(x-x_1)$$
$$y-0 = 1\cdot(x-2)$$
$$y = x-2$$

Ex A line passes through $(-4,1)$ and $(2,6)$.

Find an equation for it...

~~slope-intercept~~ ? ? ?

point-slope ? ✓ ?

$$y - y_1 = m(x - x_1)$$

$$y - 1 = m(x - (-4))$$

$$y - 1 = m(x + 4)$$

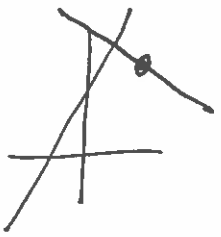
$$m = \frac{6-1}{2-(-4)} = \frac{5}{6}$$

$$y - 1 = \frac{5}{6}(x+4)$$ Done ✓

Ex One line has equation $y = 4x + 10$.

Another line is perpendicular to the first line that passes through $(2,7)$.

Find an equation for the second line.



point-slope ✓ ?

~~slope-intercept~~ ? ?

$$y - y_1 = m(x - x_1)$$

$y_1 = 7$, $x_1 = 2$, $m = ?$

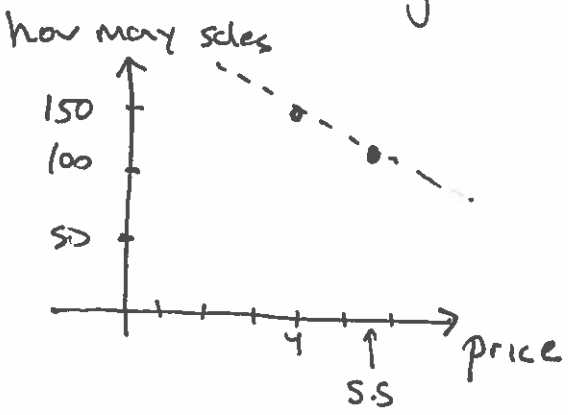
$$y - 7 = -\frac{1}{4}(x - 2)$$

$-\frac{1}{4}$ b.c. must be neg reciprocal of 4.

Ex A food cart vendor.

If we charge $4 for a burrito, they sell about 150 burritos per day.

If we chage $5.50, they only sell 120.



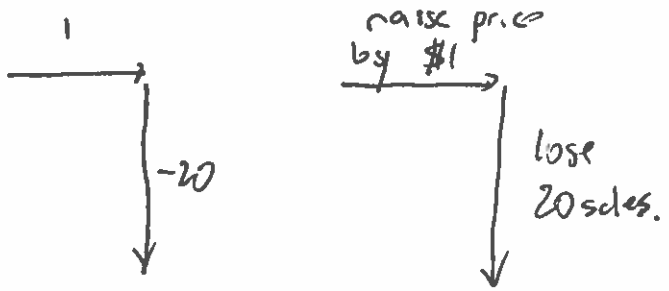
Assuming a linear relationship, write an equation for relating price to # of sales.

points: (4, 150) & (5.5, 120)

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{150 - 120}{4 - 5.5} = \frac{30}{-1.5}$$

(150 − 120: burritos; 4 − 5.5: dollars)

$$\text{slope} = -20 \ \frac{\text{burritos}}{\$}$$

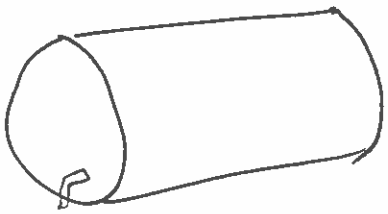


$$y - y_1 = m(x - x_1)$$

$$y - 150 = -20(x - 4)$$ Done

$$\#\text{burritos} - 150 = -20(\text{price} - 4)$$

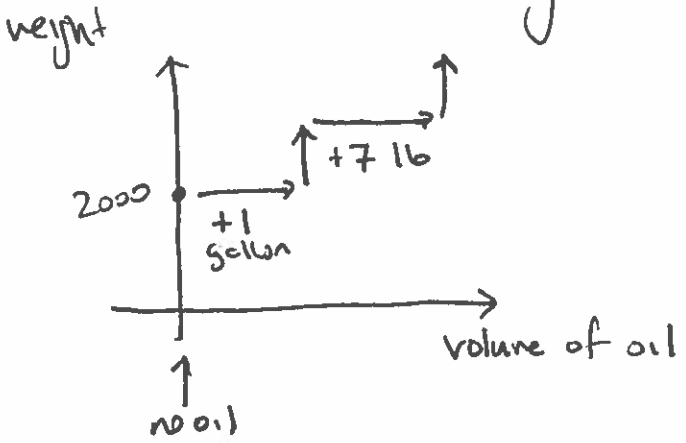
Ex A metal tank with oil ~~[illegible]~~ is to be lifted by a crane.



If empty, it weighs 2000 lb. → (0, 2000)

Each gallon of oil weighs 7 lb. → $7 \frac{lb.}{gallon}$

Write an equation modeling number of gallons of oil and how it relates to weight.



7 is the slope!

Slope-intercept ✓ ✓ ~~point-slope~~

$y = mx + b$

$y = 7x + 2000$ Done!

weight = 7 · volume + 2000

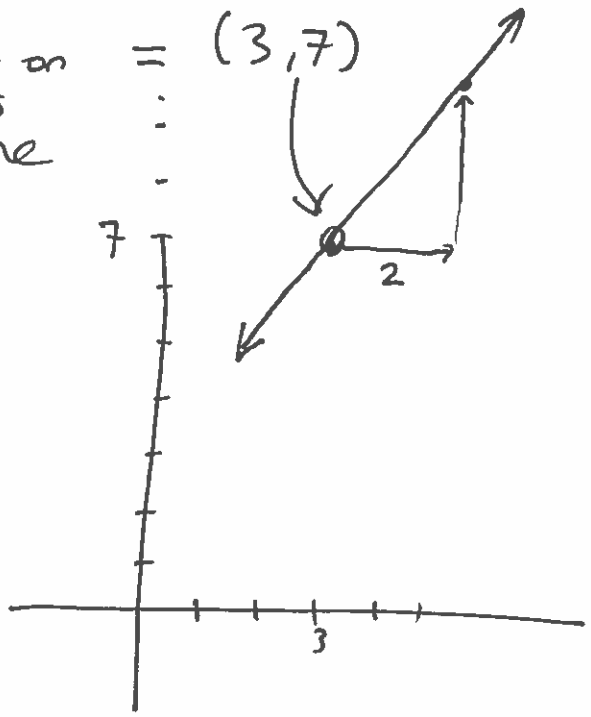
Ex $y - 7 = \frac{3}{2}(x-3)$

Sketch this line.

What can you see here?

Slope $= \frac{3}{2}$ ← rise, ← run

one point on this line $= (3,7)$



Ex Sketch $y + 2 = 3(x-4)$

slope

point $(4,-2)$