2.5 Introduction to Problem Solving (Practice with Word Problems)

let letters represent numbers.

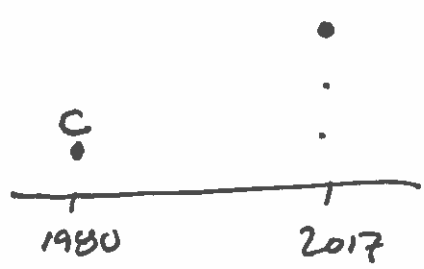
1. Bert is three inches taller than Ernie. Let e represent Ernie's height. Write an expression for Bert's height.

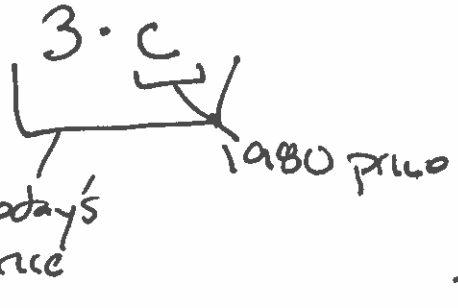
(+3 over "three inches taller"; e over "Ernie")

$e + 3$

translating English into math

2. The price of a Snickers has tripled since 1980. Let c be the price in 1980. Write an expression for today's price.





3. [The temperature in °F is 32 more than $\frac{9}{5}$ of the temperature in °C.] Let c be the temperature in Celcius. Find an expression for the °F temp.

(+32 over "32 more"; $\frac{9}{5}\cdot$ over "$\frac{9}{5}$ of")

$\frac{9}{5}c + 32$

$\frac{9}{5}$ of c, 32 more than that

Ex Bert is three inches taller than Ernie.

If Bert is 50 inches tall, how tall is Ernie?

1) Identify the variable we need to solve for and give it a name: Ernie's height: e

2) Look for "=" sign somewhere in the background.

Come up with an expression using variable e for Bert's height

$e + 3 = 50$

Bert's height in terms of "e"

Bert's height, known

3) Solve using algebra

$e + 3 - 3 = 50 - 3$

$e = 47$

4) Check: 47 satisfies the equation

$47 + 3 = 50$ ✓

Does the number make sense? Could Ernie be 47 inches?

Yes

5) Write a conclusion.

Ernie is 47 inches tall.

Ex Nora ate two fewer pancakes than Dora.

If Nora ate 7, how many did Dora eat?

1) Identify what we must solve for

how many Dora ate = D

2) Set up equation.

Look for "=" sign.

$7 = D - 2$

3) $7 + 2 = D - 2 + 2$

$9 = D$

4) Check

$7 = D - 2$

$7 = 9 - 2$

✓

Check in context

Does 9 pancakes make sense?

Sure

5) Dora ate 9 pancakes.

Ex The price of a Snickers (has) tripled since 1980. Today it costs 90¢. Back in 1980 what was the cost?

(90 = 3·c)

1) 1980 cost: c

2) $90 = 3 \cdot c$ ← attack Mult by 3

3) $\frac{90}{3} = \frac{3c}{3}$

$30 = c$

4) check?
$90 = 3(c)$
$90 = 3(30)$
✓

Is it reasonable
30¢ is the
answer?
Yes

5) [Back in 1980, a Snickers cost 30¢.

Ex [Keith] [has read] [three fourths of his summer reading list.] (6 = $\frac{3}{4} \cdot B$)
If Keith read 6 books, how many were on his list?

1) number on list =: B

2) $6 = \frac{3}{4} \cdot B$ "undo" mult by $\frac{3}{4}$

$\frac{4}{3} \cdot \frac{6}{1} = \frac{24}{3}$
$= 8$

3) $\frac{4}{3} \cdot 6 = \frac{4}{3}\left(\frac{3}{4} \cdot B\right)$

$8 = B$

4) Check
$6 = \frac{3}{4}(8)$
✓

Does 8 books
make sense?
Yes

5) Keith's list has 8 books.

Ex $\overbrace{\text{The temp. in °F}}^{98.6}$ is $\overbrace{32 \text{ more than } \frac{9}{5} \text{ of the °C temp.}}^{\frac{9}{5}\cdot C + 32}$ Normal human body temp. is 98.6°F. What is normal human body temp in °C?

1) Assign $C :=$ Celsius human body temp.

2) $98.6 = \frac{9}{5}C + 32$

3) $98.6 - 32 = \frac{9}{5}C + 32 - 32$

$66.6 = \frac{9}{5}C$

$\frac{5}{9} \cdot 66.6 = \frac{5}{9}\left(\frac{9}{5}C\right)$

Calculator

$37 = C$

4) Check $98.6 = \frac{9}{5}(37) + 32$

CALCULATOR ✓

Is 37 a reasonable answer?

∴ Sure...

5) So human body temperature is 37°C.

Ex Chelsea's apartment building has two less than triple the number of smoke detectors that Marie's house has. If Chelsea's building has 16 smoke detectors, how many does Marie's have?

1) $M :=$ # smoke detectors in Marie's house

2) $16 = 3 \cdot M - 2$

3) $16 + 2 = 3M - 2 + 2$

$18 = 3M$

$\frac{18}{3} = \frac{3M}{3}$

$6 = M$

4) Check:

$16 = 3(6) - 2$

✓

Is 6 smoke detectors reasonable? Yes

5) So Marie's house has 6 smoke detectors.

Ex Denise buys pumpkins for $8 each, and spends another $5 on a pumpkin carving kit. If she spends $53 total, how many pumpkins did she buy?

1) Let $p :=$ # pumpkins bought

2) $53 = \underbrace{8p}_{\text{cost of all those pumpkins}} + 5$

3)
$$53 = 8p + 5$$
$$53 - 5 = 8p + 5 - 5$$
$$48 = 8p$$
$$\frac{48}{8} = \frac{8p}{8}$$
$$6 = p$$

4) $53 = 8(6) + 5$ ✓

Is 6 pumpkins reasonable?

Yes

5) Denise bought 6 pumpkins.

Ex Valerie is making grilled cheese sandwiches. The number of bread slices she has (17) is one more than twice the number of cheese slices she has. If she has 17 slices of bread, how many slices of cheese does she have?

1) $s :=$ # slices of cheese

2) $17 = 2 \cdot s + 1$

3) $17 - 1 = 2s + 1 - 1$

$16 = 2s$

$\frac{16}{2} = \frac{2s}{s}$

$8 = s$

4) Check

$17 = 2(8) + 1$ ✓

Is 8 slices reasonable? ✓

5) Valerie has 8 slices of cheese.

Ex Mindy has moved half as many pieces of firewood as Cindy has. Together they moved 120 pieces. How many has each moved?

↳ Looking for two answers.

1) Assign variable to what we're looking for

Other thing:

$C :=$ how much Cindy moved — how much Mindy moved $\frac{C}{2}$

use C to write an expression

2) $\underbrace{C + \frac{C}{2}}_{\text{together...}} = \underbrace{120}_{\text{moved 120}}$

3) $2\left(C + \frac{C}{2}\right) = 2(120)$

$2C + 2 \cdot \frac{C}{2} = 240$

$2C + C = 240$

$3C = 240$

$\frac{3C}{3} = \frac{240}{3}$

$C = 80$

4) $80 + \frac{80}{2} \stackrel{?}{=} 120$ ✓

Is 80 pieces reasonable?

5) Cindy moved 80 pieces, and Mindy moved 40.

Ex Dad gets one fewer than twice as many cookies as Henry. If there are 8 cookies, how many do Dad, Henry get?

↓

two things

1) $H :=$ how many Henry eats

how many Dad eats? $2H-1$

$2 \cdot H - 1$

2) $H + (2H-1) = 8$

3)
$$\begin{aligned} 3H - 1 &= 9 \\ 3H - 1 + 1 &= 8 + 1 \\ 3H &= 9 \\ \frac{3H}{3} &= \frac{9}{3} \\ H &= 3 \end{aligned}$$

4) $3 + (2(3)-1) \stackrel{?}{=} 8$

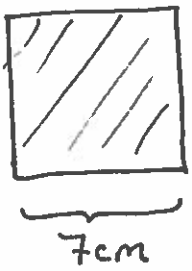
✓

Is 3 cookies reasonable for Henry?

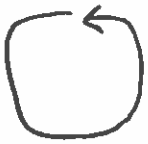
$\Longrightarrow$ Dad ate 5...

5) Dad ate 5 cookies, Henry ate 3.

Geometry

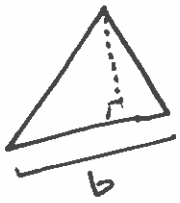
square 

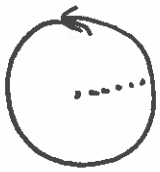
Perimeter of a square = $7cm \cdot 4$
$= 28cm.$



Area of square = $(7cm)(7cm)$
$= 49 cm^2$

rectangle 





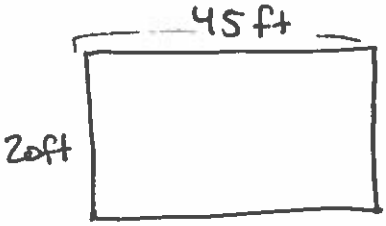
perimeter is distance around

area

$A = b \cdot h$ $\quad A = \frac{1}{2} b \cdot h$ $\quad A = \pi \cdot r^2$

Ex A family puts fencing around their yard, that's 45ft by 20ft. How much fencing do they need?

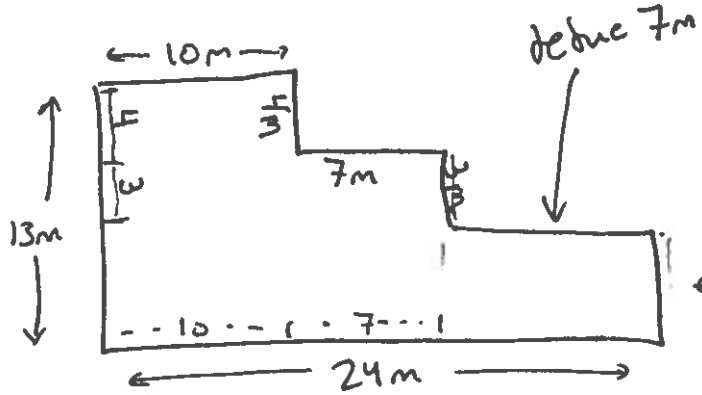
Draw!



Fencing = perimeter

$45ft + 20ft + 45ft + 20ft$
$= 65ft + 65ft$
$= 130ft.$

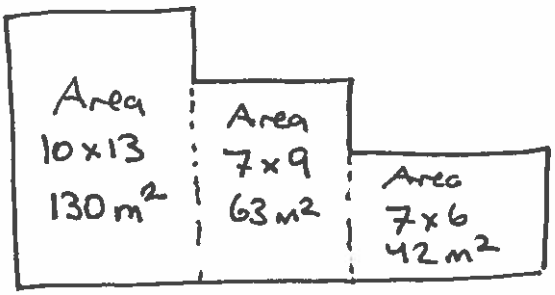
Ex



What's the perimeter?

13 + 10 + 4 + 7 + 3 + 7 + 6 + 24

= 74m

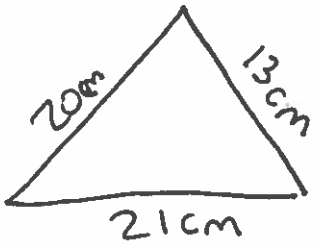


How much area?

$130m^2 + 63m^2 + 42m^2$

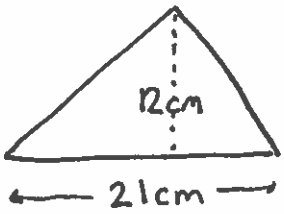
$= 235m^2$

Ex



perimeter = 20cm + 13cm + 21cm

= 54cm



area?

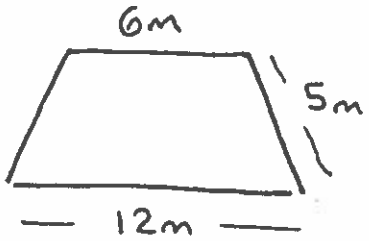
$A = \frac{1}{2} \cdot bh$

$= \frac{1}{2}(21cm)(12cm)$

$= (21cm)(6cm)$

$= 126\ cm^2$

Trapezoid

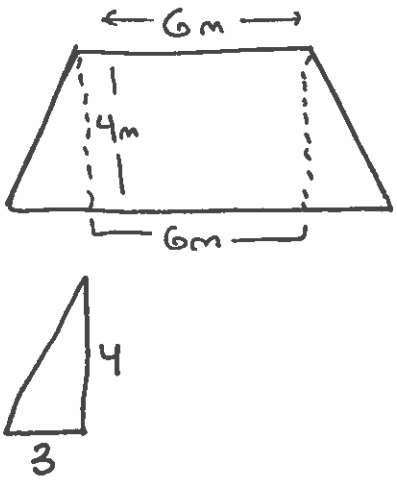


Perimeter

$6m + 5m + 12m + 5m$

$= 11m + 17m$

$= 28m$



Area

Rectangle: $(6m)(4m)$

$24m^2$

One Triangle:

$\frac{1}{2}(3m)(4m)$

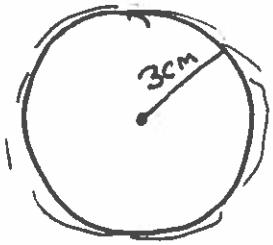
$= (3m)(2m)$

$= 6m^2$

All: $24m^2 + 6m^2 + 6m^2$

$= 36m^2$

Circles:



perimeter (circumference)

$\boxed{C = 2\pi r}$

$= 2 \cdot \pi \cdot (3cm)$

3.1415926....

$C \approx 18.8\_\_ cm$

$2.0 * \pi * 3$

area

$\boxed{A = \pi r^2}$

$A = \pi (3cm)^2$

$= \pi \cdot 9 \cdot cm^2$

$\approx 28.27\_\_ cm^2$

Ex A triangle has three sides. The longest side is triple the shortest side. The medium side is one less than the longest side. The perimeter is 30 in. How long are the sides?

→ three things

1) L = longest side

medium side = $L-1$

shortest side = $\frac{L}{3}$

2) $L + (L-1) + \frac{L}{3} = 30$

3) $2L - 1 + \frac{L}{3} = 30$

$3\left(2L - 1 + \frac{L}{3}\right) = 3(30)$

$6L - 3 + L = 90$

$7L - 3 = 90$

$7L - 3 + 3 = 90 + 3$

$7L = 93$

$\frac{7L}{7} = \frac{93}{7}$

$L = 93/7.$

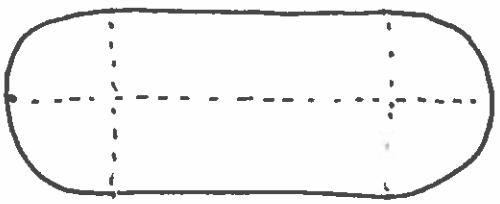
4) $\underbrace{\frac{93}{7} + \left(\frac{93}{7} - 1\right) + \frac{93/7}{3}}_{\text{CALC}} = 30$ ✓

Reasonable?

$\frac{93}{7} \approx 13.2\ldots$

5) The longest side is $93/7$, the shortest side is $\frac{1}{3}\frac{93}{7} = \frac{31}{7}$, medium side is $\frac{93}{7} - 1$, $\frac{93}{7} - \frac{7}{7}$, $\frac{86}{7}$.

Ex A skating rink has total perimeter 1000 m.



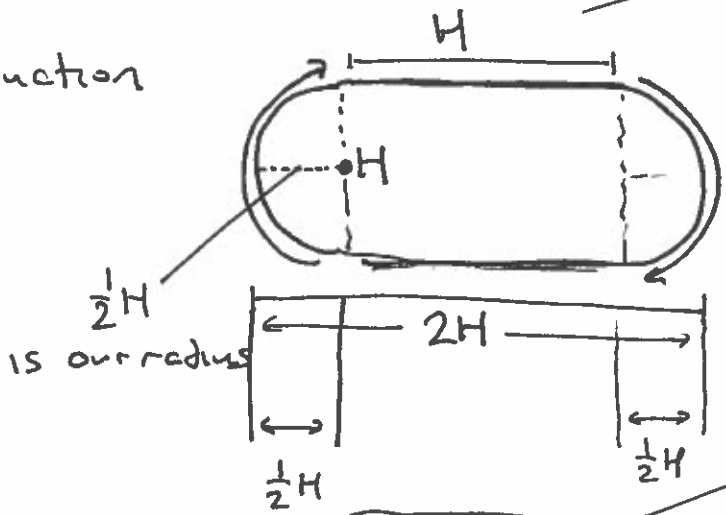
Its width is twice its height.

What are the dimensions of this skating rink?

1) $H :=$ height & width $= 2 \cdot H$

(no-no to use W)

2) Set up equation



$2 \cdot \pi \cdot \frac{1}{2} \cdot H$

$2 \cdot \frac{1}{2} \cdot \pi \cdot H$

$1 \cdot \pi \cdot H$

$\pi \cdot H$

$$1000 = \underbrace{H}_{\text{top}} + \underbrace{H}_{\text{bottom}} + \underbrace{2\pi\left(\tfrac{1}{2}H\right)}_{\text{two sides make a full circle}}$$

3) $$1000 = 2H + \pi H$$

$$1000 = (2 + \pi) \cdot H$$

$$\frac{1000}{2+\pi} = \frac{(2+\pi) \cdot H}{(2+\pi)}$$

$$\frac{1000}{2+\pi} = H \approx 194.5$$

5) the rink is 194.5 m high by $2 \cdot (194.5)$ wide