2.4 (last section for next week's test)

"Percent" means "out of 100"

Century
Centipede
centimeter
one cent
} — 100 (C in Roman)

To convert a regular decimal to a percentage:

Ex 0.15 (decimal) "fifteen hundredths"

$\frac{15}{100} = 15\%$ (percentage) ← percent symbol

Ex 0.6 (decimal)

$0.6 \cdot \frac{100}{100}$

$= \underline{0.6 \cdot 100} \cdot \frac{1}{100}$

$= 060. \cdot \frac{1}{100}$

$= 60\ \%$

0.6 → slide decimal 2 places and introduce % sign = 60. %

Ex 0.0006 (decimal) → 00.06 %

0.06 %

Think about sizes.

0.6 — correctly → 60%

0.6 — incorrectly → 6%

Ex 0.93 (decimal) $= 93\%$

Ex 1.08 (decimal) $= 108\%$ (both bigger than 1)

correct

incorrect → 0.0108% ~~0.0108%~~ super small

Ex $2 = 2 \cdot \frac{100}{100}$

$= 2 \cdot 100 \cdot \frac{1}{100}$

$= 200\%$

very large

Ex 0.003 (decimal) $= 000.3\%$

$= 0.3\%$

Convert percentage to a decimal:

Ex $25\% = 25 \cdot \frac{1}{100} = \frac{25}{100} = 0.25$

25% → strip "%" away → decimal 2 left → 0.25

Ex $16\% = 0.16$

Ex $320\% = 320$

$= 3.2$

Ex $3.06\% = 0.0306$

$= 0.0306$

$3.06\% = 3.06 \cdot \frac{1}{100}$

$= 0.0306$

$$A = P \cdot B$$

- A: amount (that you finish with)
- P: percent (converted to a decimal)
- B: base (amount you start with)

Ex What is 37% of 42?

"of" → multiplication

$A = P \cdot B$

$A = 0.37 \cdot B$

$A = 0.37\,(42)$

$A = 15.54$

How to use?

Had 42...
Calculated 37% of that...
Got...

$$\begin{array}{r} {}^{1}42 \\ \times .37 \\ \hline 294 \\ 126 \\ \hline 15.54 \end{array}$$

Ex What percent of 30 is 6?

$P \cdot 30 = 6$

(what percent → P; of → $\cdot$; is → $=$)

$A = P \cdot B$

$6 = ? \cdot 30$

$6 = P \cdot 30$ ← into mult.

$\Rightarrow \frac{6}{30} = \frac{P \cdot 30}{30}$

$\Rightarrow \frac{1}{5} = P \Rightarrow 0.2 = P$

So 20% of 30 is 6.

Memorize

$\frac{1}{2} = 0.5$

$\frac{1}{3} = 0.333\ldots$

$\frac{1}{4} = 0.25$

$\frac{1}{5} = 0.2$

$\frac{1}{6} = 0.1666\ldots$

Ex 24% of what number is 70?

$A = P \cdot B$ (B = ?)

$70 = 0.24 \cdot B$

result after doing some percentage calculations

undo mult by 0.24

$\frac{70}{0.24} = \frac{0.24B}{0.24}$

$291.666... = B$

$291.6 = B$

291.7

So 24% of 291.7 is 70.

Apps Stewart has an annual salary of $36,000. Automatic 2.5% raise every year.

What will his raise be? (in dollars)

What will his salary be after raise?

$A = P \cdot B$

$A = 0.025 \cdot B$

36,000 is what Stewart starts with

$A = 0.025(36000)$

$A = 900$

↳ what does this mean? This is an additional $900 each year.

And so the salary will be $36,900.

Ex Tammy buys designer jeans in Iowa, and is charged $5.94 in sales tax. The retail price (before tax) is $99.

What is the total receipt cost?

$104.94

What is the sales tax percentage in Iowa?

$A = P \cdot B$ (B: base)

$A = P \cdot 99$ (P: percent(?))

after-amount

tax amount or (total receipt cost) would be ok too

$104.94 = P \cdot 99$

$\frac{104.94}{99} = \frac{P \cdot 99}{99}$

$1.06 = P$

this

$106\% = P$

what does this mean in context?

100% (jeans price) + 6% (sales tax)

The sales tax in Iowa is 6%.

Ex A pair of roundtrip Amtrak tickets totals $718. A discount of 10% (0.1) is applied.

How much is the discount? What is the final purchase price?

$A = P \cdot B$ ← base. before the discount, $718

$A = 0.1 \cdot B$

$A = \frac{0.1}{\text{divide by 10}}(718)$

$A = 71.8$ → Saved $71.80

Final purchase price: will be $646.20.

$$\begin{array}{r} 6\ 11\ 7 \\ \cancel{7}\cancel{1}\cancel{8}.00 \\ -\ 71.80 \\ \hline 646.20 \end{array}$$

Ex A flatscreen TV is on sale for $1699.99. The orgind price was $2299.99. What was the discount %? (How much off?)

$A = P \cdot B$

after ← $1699.99 = P \cdot 2299.99$

$\frac{1699.99}{2299.99} = P$

$0.7391 = P$ → $P = 73.91\%$

You paid 73.91% of orgind.

$100\% - 73.91\% = 26.09\%$

So, its a 26.09% off sale.

Ex An 18% (0.18) tip will be added to a dinner that was $157.50. How much ($) are you tipping? What's the total?

percentages only go here for P.

$$A = P \cdot B$$

after... we will know how much to tip

before... any calcs the bill is known to be $157.50

$$A = 0.18(157.50)$$

$$A = 28.35$$

after (the tip calculated) we will tip $28.35.

$$\begin{array}{r} 157.50 \\ 28.35 \\ \hline 185.85 \end{array}$$

So the total is $185.85.

Ex Gas went from $2.59 (before) to $2.83 (after) one month. What percentage increase was that?

$$A = P \cdot B$$

$$2.83 = P \cdot 2.59$$

$$\frac{2.83}{2.59} = P$$

$$1.09266\ldots = P$$

$$109.26\ldots\% = P$$

100% of what it was + 9.266% extra

Gas rose by 9.266%.

Ex Gas declined from $3.08 (before) to $2.71 (after) one month. By what % did it drop?

$$A = P \cdot B$$

$$2.71 = P\,(3.09)$$

$$\frac{2.71}{3.09} = P$$

$$0.87987\ldots = P$$

$$87.987\% = P$$

You're paying 87.987% of last month's... decreased by $100\% - 87.987\% = 12.013\%$

2.4 continued

Also about formulas (like $A = P \cdot B$)

$$A = P \cdot B$$

(if I knew P, B, quickly compute A)

try to isolate P (undo mult by B)

$$\frac{A}{B} = \frac{PB}{B}$$

$$\frac{A}{B} = P$$

$$P = \frac{A}{B}$$

(If I knew A, B, quickly compute P.)

Take formulas and isolate a specific variable.

Ex $d = r \cdot t$ (distance, rate (speed), time)

Solve for t.

into this...

$$d = r\boxed{t}$$

$$\Longrightarrow \frac{d}{r} = \frac{rt}{r}$$

$$\Longrightarrow \frac{d}{r} = t$$

$$\Longrightarrow t = \frac{d}{r}$$

Ex $I = Prt$ (interest (in $), principal, interest rate, time elapsed) Solve for t.

$$I = Pr\boxed{t}$$ peel away P, r

$$\frac{I}{Pr} = \frac{\cancel{P}\cancel{r}t}{\cancel{P}\cancel{r}}$$

$$\frac{I}{Pr} = t$$

$$t = \frac{I}{Pr}$$

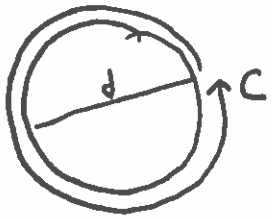
How long until an interest payment?

next account

$I = \$500$
$P = \$100000$
$r = 0.06$

$I = \$1200$
$P = \$200000$
$r = 0.07$

Ex $C = \pi d$ (circumference, diameter)



$3.14159265\ldots$ [calculator button π]

Solve for d

$C = \pi \boxed{d}$

$\Rightarrow \frac{C}{\pi} = \frac{\pi \boxed{d}}{\pi}$

$\frac{C}{\pi} = d$

$d = \frac{C}{\pi}$ ("pi")

Ex $y = mx + b$ Solve for x.

$y = m\boxed{x} + b$

"$x = y - b / m$" is like $x = y - \frac{b}{m}$ — Should enter "$x = (y-b)/m$"

$\Rightarrow y - b = m\boxed{x} + b - b$

$y - b = m\boxed{x}$

$\Rightarrow \frac{y-b}{m} = \frac{m\boxed{x}}{m}$

$\Rightarrow \frac{y-b}{m} = x$

$x = \frac{y-b}{m}$

Ex $F = \frac{9}{5}C + 32$

°Fahrenheit, °Celsius

Solve for C. $F = \frac{9}{5}C + 32$

$\Rightarrow F - 32 = \frac{9}{5}C + 32 - 32$

$\Rightarrow F - 32 = \frac{9}{5}C$

to undo $\frac{9}{5}$

$\Rightarrow \frac{5}{9}(F-32) = \frac{5}{9}\cdot\frac{9}{5}C$

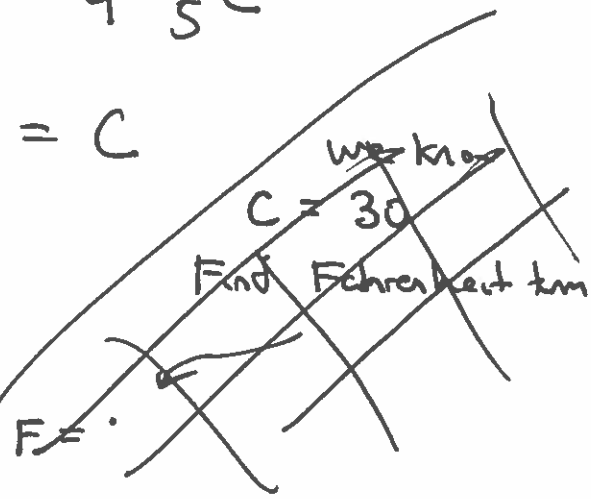
$\Rightarrow \frac{5}{9}(F-32) = C$

So $C = \frac{5}{9}(F-32)$.

OR

$C = \frac{5}{9}F - \frac{5}{9}\cdot 32$

$C = \frac{5}{9}F - \frac{160}{9}$



We know its 81° F. What's the Celsius temp?

new formula

$F = \frac{9}{5}C + 32$

$81 = \frac{9}{5}C + 32$

$81 - 32 = \frac{9}{5}C$

$49 = \frac{9}{5}C$

$\frac{5}{9}\cdot 49 = C$

$\frac{245}{9} = C$

$27.2\ldots = C$

$C = \frac{5}{9}(F-32)$

$C = \frac{5}{9}(81-32)$

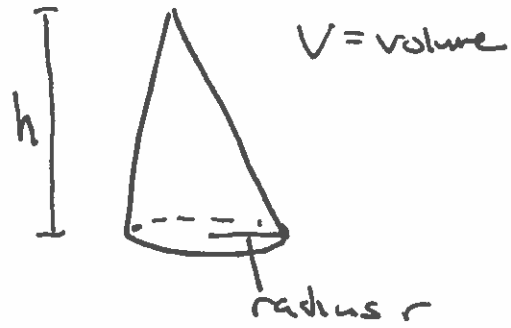
$= \frac{5}{9}(49)$

$= \frac{245}{9}$

$= 27.2\ldots$

Ex $V = \pi r^2 h$

Solve for h.



$V = \pi r^2 \boxed{h}$

h is multiplied by $\pi \cdot r^2$, so undo that!

$\Rightarrow \frac{V}{\pi r^2} = \frac{\pi r^2 \boxed{h}}{\pi r^2}$

$\frac{V}{\pi r^2} = h$ → $h = \frac{V}{\pi r^2}$

"h = V / pi r^2"

(div, mult)

$h = \left(\frac{V}{\pi}\right) \cdot r^2$!!!

"h = V/(pi r^2)" ✓

Ex $m = \frac{a+b+c}{3}$ (average of three things)

Solve for c.

$m = \frac{a+b+\boxed{c}}{3}$

$\Rightarrow 3m = 3\frac{a+b+c}{3}$ → $3m = a+b+\boxed{c}$

$3m - a = a+b+c-a$

$3m - a = b+\boxed{c}$

$3m-a-b = b+c-b$

$3m-a-b = c$

So $\boxed{c = 3m-a-b}$