

- 1) Simplify each side of a linear equation
- 2) Collect variable terms on left,
non-variable terms on right
 ↓
 add or subtract on both sides
- 3) Isolate variable
 ↓
 Multiply or divide (left with $x = \#$)
- 4) Check.

Ex Emil has \$1000 in savings,
and \$60 each month. Jackie has
\$2200 in savings and saves \$30 each
month. When will their accounts equal?

- ① Make variable
- | | |
|----------|-----|
| "when" | w |
| "time" | t |
| "months" | m ← |
| generic | x |

② "to be"

③ Use words is OK

Emil's balance after m months = Jackie's balance after m months

$$1000 + 60 \cdot m = 2200 + 30 \cdot m$$

④ translate words into symbols

$$1000 + 60m - 30m = 2200 + 30m - 30m$$

$$1000 + 30m = 2200$$

$$1000 + 30m - 1000 = 2200 - 1000$$

$$30m = 1200$$

$$\frac{30m}{30} = \frac{1200}{30} \rightarrow m = 40$$

1) nothing to do

2)

3)

$$4) \quad 1000 + 60(\quad) = 2200 + 30(\quad)$$

$$1000 + 60(40) \stackrel{?}{=} 2200 + 30(40)$$

$$1000 + 2400 \stackrel{?}{=} 2200 + 1200$$

$$3400 \stackrel{?}{=} 3400 \quad \checkmark$$

3 years & 4 months

So after 40 months, their balances equal out (at \$3400).

Ex Solve $7x - (3x - 1) = 5$ $7x - (3x + (-1)) = 5$

subtract all this

1) Simplify $7x - 3x - (-1) = 5$

$$4x + 1 = 5$$

2)

$$4x + 1 - 1 = 5 - 1$$

$$4x = 4$$

3) $x = 1$

4)

$$7(\quad) - (3(\quad) - 1) = 5$$

$$7(1) - (3(1) - 1) \stackrel{?}{=} 5$$

$$7 - (3 - 1) \stackrel{?}{=} 5$$

$$7 - 2 \stackrel{?}{=} 5$$

$$5 = 5 \quad \checkmark$$

Ex Solve $(y-3) - (y+7) = 4y$

OK to drop these they are redundant

$$y-3 - (y+7) = 4y$$

$$\underline{y}-3 - \underline{y}-7 = 4y$$

$$-10 = 4y$$

$$\frac{-10}{4} = \frac{4y}{4}$$

$$-\frac{5}{2} = y$$

$$y = -\frac{5}{2}$$

$$\left(-\frac{5}{2} - 3\right) - \left(-\frac{5}{2} + 7\right) = 4 \cdot \left(-\frac{5}{2}\right)$$

$$\left(-\frac{5}{2} - \frac{3}{1}\right) - \left(-\frac{5}{2} + \frac{7}{1}\right) = \frac{4^2}{1} \cdot \left(-\frac{5}{2}\right)$$

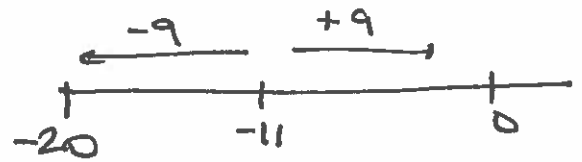
$$\left(-\frac{5}{2} - \frac{6}{2}\right) - \left(-\frac{5}{2} + \frac{14}{2}\right) = -10$$

$$-\frac{11}{2} - \left(\frac{9}{2}\right) \stackrel{?}{=} -10$$

$$\frac{-20}{2} \stackrel{?}{=} -10$$

$$-\frac{11}{2} + \left(-\frac{9}{2}\right)$$

$$\frac{-20}{2}$$



Ex Solve

$$4(3+x) = 5(x+1) + 7 + x$$

$$12 + 4x = 5x + 5 + 7 + x$$

$$12 + 4x = 6x + 12$$

$$12 + 4x - 6x = 6x + 12 - 6x$$

$$12 - 2x = 12$$

$$12 - 2x - 12 = 12 - 12$$

$$-2x = 0 \Rightarrow x = \frac{0}{-2}$$

$$\Rightarrow x = 0$$

$$4(3+0) \stackrel{?}{=} 5(0+1) + 7 + 0$$

$$4(3) = 5(1) + 7$$

$$12 = 5 + 7$$

✓

Step 1: Simplify the algebraic expression on each side.

Step 2: Collect all variable terms on one side and all constant terms on the other side.

Step 3: Isolate the variable and solve.

Step 4: Check the proposed solution in the **original** equation.

#1-12: Solve each equation algebraically and check the solution.

1. $2x + 3x = -35$

$$5x = -35$$

$$\frac{5x}{5} = \frac{-35}{5}$$

$$x = -7$$

2. $9n - 5n = 2$

$$4n = 2$$

$$\frac{4n}{4} = \frac{2}{4}$$

$$n = \frac{1}{2}$$

3. $4a + 1 = 9(a - 1)$

$$4a + 1 = 9a - 9$$

$$4a + 1 - 9a = 9a - 9 - 9a$$

$$-5a + 1 = -9$$

$$-5a + 1 - 1 = -9 - 1$$

$$-5a = -10$$

$$\frac{-5a}{-5} = \frac{-10}{-5}$$

$$a = 2$$

4. $2(m + 5) = 3(3m + 1)$

$$2m + 10 = 9m + 3$$

$$2m + 10 - 9m = 9m + 3 - 9m$$

$$-7m + 10 = 3$$

$$-7m + 10 - 10 = 3 - 10$$

$$-7m = -7$$

$$\frac{-7m}{-7} = \frac{-7}{-7}$$

$$m = 1$$

5. $2x + 4x - 7 = 13x - x + 5$

$$6x - 7 = 12x + 5$$

$$6x - 7 - 12x = 12x + 5 - 12x$$

$$-6x - 7 = 5$$

$$-6x - 7 + 7 = 5 + 7$$

$$-6x = 12$$

$$\frac{-6x}{-6} = \frac{12}{-6}$$

$$x = -2$$

6. $3y + 4(y - 2) = 15 - (2y - 13)$

$$3y + 4y - 8 = 15 - 2y + 13$$

$$7y - 8 = 28 - 2y$$

$$7y - 8 + 2y = 28 - 2y + 2y$$

$$9y - 8 = 28$$

$$9y - 8 + 8 = 28 + 8$$

$$9y = 36$$

$$\frac{9y}{9} = \frac{36}{9}$$

$$y = 4$$

7. $1.1c + 1.4c = 20$

$$2.5c = 20$$

$$\frac{2.5c}{2.5} = \frac{20}{2.5}$$

$$c = \frac{20}{2.5} \cdot \frac{10}{10}$$

$$c = \frac{200}{25}$$

$$c = 8$$

8. $0.09x + 4.3 = 6.37$

~~$$0.09x + 4.3 =$$~~

$$0.09x + 4.3 - 4.3 = 6.37 - 4.3$$

$$0.09x = 2.07$$

$$\frac{0.09x}{0.09} = \frac{2.07}{0.09}$$

$$x = \frac{2.07}{0.09} \cdot \frac{100}{100} = \frac{207}{9}$$

$$x = 23$$

9. $1.3k + 2.7 = 2k - 2.9$

$$1.3k + 2.7 - 2k = 2k - 2.9 - 2k$$

~~$$1.3k + 2.7 =$$~~

$$-0.7k + 2.7 = -2.9$$

$$-0.7k + 2.7 - 2.7 = -2.9 - 2.7$$

$$-0.7k = -5.6$$

$$\frac{-0.7k}{-0.7} = \frac{-5.6}{-0.7}$$

$$k = \frac{-5.6}{-0.7} \cdot \frac{10}{10} = \frac{56}{7} = 8$$

10. **CALC** $6.375y - 5.21 = 20.29$

$$6.375y - 5.21 + 5.21 = 20.29 + 5.21$$

$$6.375y = 25.5$$

$$\frac{6.375y}{6.375} = \frac{25.5}{6.375}$$

$$y = \frac{25.5}{6.375}$$

$$y = 4$$

$$\begin{array}{r} 20.29 \\ + 5.21 \\ \hline 25.50 \end{array}$$

11. CALC $2.3h = h - 2.34$

12. CALC $1.05n - 3.8 = 0.4n + 5.95$

For #24-32, Write an equation, including units, to represent the situation. Then solve your equation and state your answer. Then check the solution in the original wording of the problem.

24. Ms. Pac-Man has 9,870 points, and she needs a total of 10,000 points to earn a free life. Each dot she eats is worth 10 points. How many dots does she need to eat to earn a free life? (Let d be the number of dots.)

①

② d

③ Points she has = 10000

④ $9,870 + d \cdot 10 = 10000$

25. Brit spent \$163 buying 4 concert tickets, which included a one-time "convenience fee" of \$7. What was the cost of each ticket? (Let t be the cost of each ticket.)

① t

② convenience fee & ticket costs = 163

$7 + 4 \cdot t = 163$

$\frac{163 - 7}{4} = t$

26. When Chantal goes for a bike ride, it takes her 10 minutes to get ready, and then she rides at an average speed of 3 minutes per mile. If her last bike ride, including the time to get ready, took 55 minutes, how far did she ride? (Let d be the distance she rode, in miles.)

①

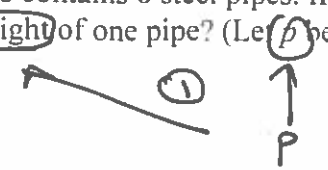
② Chantal's time ... = 55

$10 + 3d = 55$ (Ready to solve.)

27. A crate that weighs 25 pounds contains 8 steel pipes. If the total weight of the crate and pipes is 45 pounds, what is the weight of one pipe? (Let p be the weight of one pipe.)

②
=

③ combined weight of crate & pipes = 45



Draw picture!

④ $25 + 8p = 45$ (Now solve.)

28. Andre has \$67 saved towards a new X-Station, a video game system that costs \$199. If he does odd jobs for \$12 per hour to earn the rest of the money, how many hours must he work? (Let h be the number of hours.)

29. The copy shop Stapling Max charges \$0.03 per page printed. Donita has a document printed, then she pays \$1.90 to have her document bound with a cover. She pays a total of \$3.46. How many pages long is her document? (Let p be the number of pages.)

~~1.90 + 0.03p = 3.46~~

①

②

$1.90 + 0.03p = 3.46$

one binding

price per page.

pages

Ex Solve $\frac{1}{2}x + \frac{3}{4} = \frac{2}{3}x$ least common multiple

If you do:

$$\frac{1}{2}x + \frac{3}{4} - \frac{2}{3}x = \frac{2}{3}x - \frac{2}{3}x$$

Combine! \rightarrow

$$= 0$$

Step 0: Identify LCM of denominators

Denoms: 2, 4, 3

LCM is 12 \leftarrow
Multiply each side by

$$\frac{1}{2}x + \frac{3}{4} = \frac{2}{3}x$$

$$12 \left(\frac{1}{2}x + \frac{3}{4} \right) = 12 \left(\frac{2}{3}x \right)$$

$$12 \left(\frac{1}{2}x \right) + 12 \left(\frac{3}{4} \right) = 12 \left(\frac{2}{3}x \right)$$

$$6x + 9 = 8x$$

1) Nothing to do

2) $6x + 9 - 8x = 8x - 8x$

$$\begin{array}{r} + 9 - 8x \\ -2x \\ \hline -2x + 9 \end{array} = 0$$

$$-2x + 9 - 9 = 0 - 9$$

$$-2x = -9$$

3) $\frac{-2x}{-2} = \frac{-9}{-2}$

$$x = \frac{9}{2}$$

$$\underline{\text{Ex}} \quad \frac{2x}{5} - \frac{x}{3} = 3$$

$$15 \left(\frac{2x}{5} - \frac{x}{3} \right) = 15(3)$$

$$15 \left(\frac{2x}{5} \right) - 15 \left(\frac{x}{3} \right) = 15(3)$$

$$6x - 5x = 45$$

$$x = 45$$

Want to "clear
denominators".

See 15 is LCM
of denominators.

$$\underline{\text{Ex}} \quad \frac{2x}{3} + \frac{5}{6} = \frac{3x}{2}$$

$$6 \left(\frac{2x}{3} + \frac{5}{6} \right) = 6 \left(\frac{3x}{2} \right)$$

$$6 \left(\frac{2x}{3} \right) + 6 \left(\frac{5}{6} \right) = 6 \left(\frac{3x}{2} \right)$$

$$4x + 5 = 9x$$

$$4x + 5 - \underbrace{4x} = 9x - \underbrace{4x}$$

$$5 = 5x$$

$$1 = x$$

Identify LCM
of denominators: 6

$x = 1.$

$$\underline{E-} \quad 540 + 19m + 16\left(\frac{1}{4}m\right) = 36\left(\frac{1}{2}(m-3)\right) + 15\left(\frac{1}{2}(m-3)\right)$$

multiply!
multiply!

$$540 + 19m + 4m = 18(m-3) + \frac{15}{2}(m-3)$$

$$540 + 23m = 18m - 54 + \frac{15}{2}m - \cancel{\frac{15}{2}(3)}$$

$-\frac{45}{2}$

LCM
2

$$2(540 + 23m) = 2\left(18m - 54 + \frac{15}{2}m - \frac{45}{2}\right)$$

$$1080 + 46m = 36m - 108 + 15m - 45$$

$$1080 + 46m = \underline{51m} - 153$$

$$1080 + 46m - \underline{51m} = 51m - 153 - \underline{51m}$$

$$\cancel{1080} + \underline{1080} - 5m = -153$$

$$1080 - 5m - 1080 = -153 - 1080$$

$$-5m = -1233$$

$$\frac{-5m}{-5} = \frac{-1233}{-5}$$

$$m = \frac{1233}{5}$$

$$\begin{array}{r} 1080 \\ 153 \\ \hline 1233 \end{array}$$

Still 2.3

$$\text{Solve } x + 5 = x - 2$$

↑
Attack

Has no solutions!
It's impossible
to solve this
one.

$$x + 5 - x = x - 2 - x$$

$$5 = -2$$

(Clearly false! It doesn't matter
what x is, you can't make $5 = -2$.)

This equation has no solutions.

The solution set is empty.

The solution set is \emptyset .

Symbol for
the
empty set
(not the
number zero.)

Ex

$$3x + 8 = 2(x + 4) + x$$

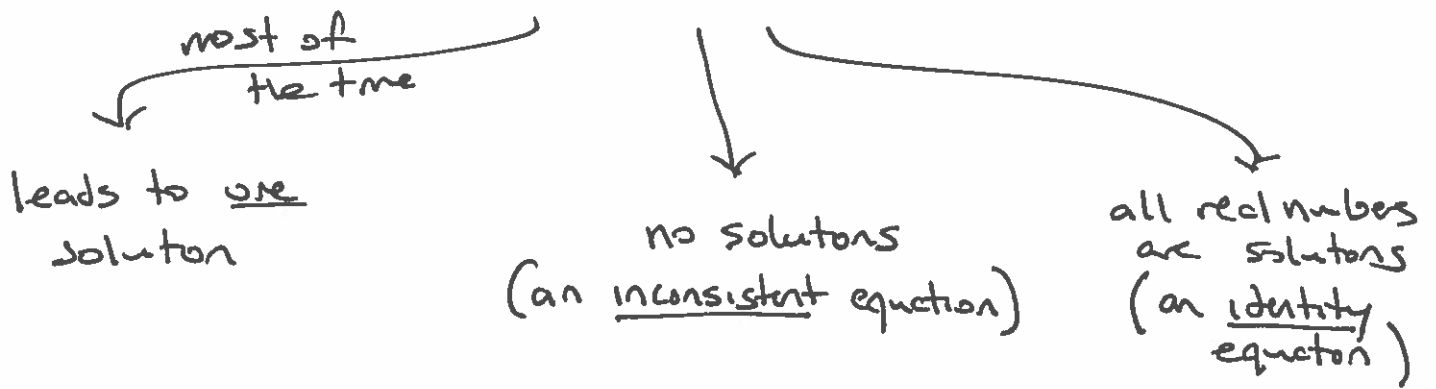
$$3x + 8 = 2x + 8 + x$$

$$3x + 8 = 3x + 8$$

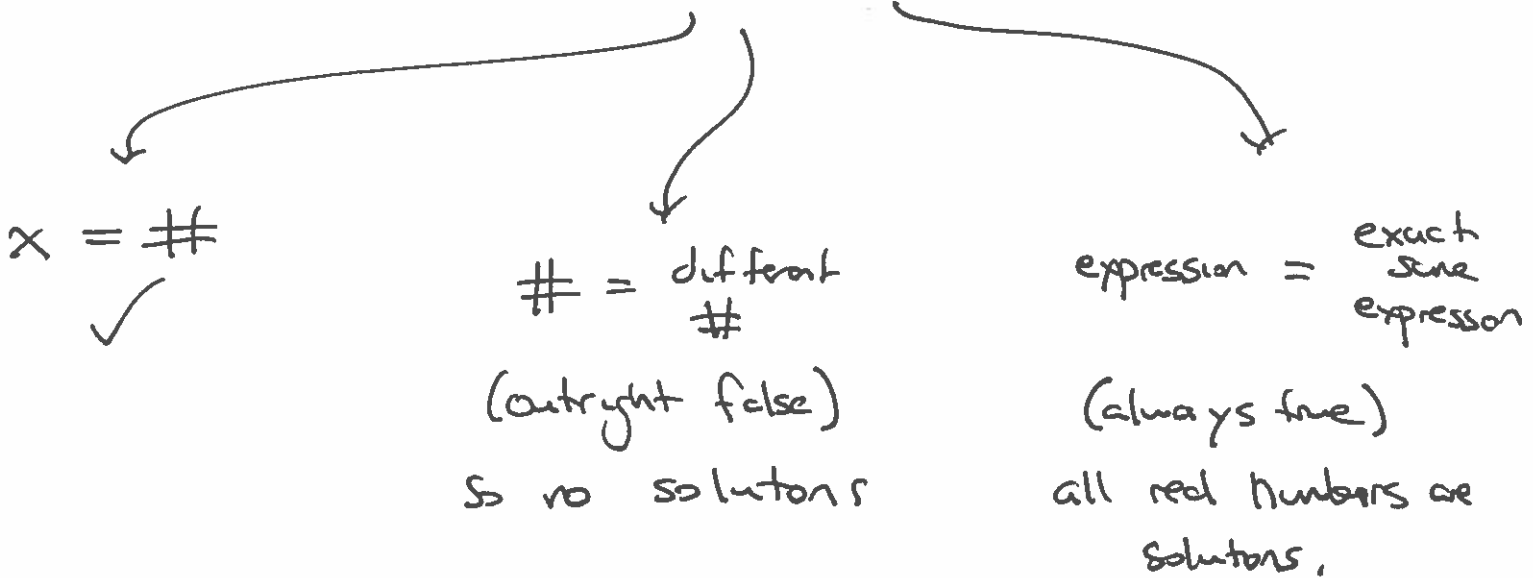
Automatically true no matter what x is.

* Solutions are all real numbers.

Linear Equation



Try to solve using our 4 steps



Ex

$$5x - 7 = 5(x-2) + 3$$
$$5x - 7 = 5x - 10 + 3$$
$$5x - 7 = 5x - 7$$

all real numbers are solutions (an identity)

Ex

$$3x + 4x + 1 = 9x - 8 - 2x$$
$$7x + 1 = 7x - 8$$

↑!!

$$7x + 1 - 7x = 7x - 8 - 7x$$
$$1 = -8$$

No solutions (an inconsistent equation)

Ex $2(3x+4) = 3(2x+3) - 1$

$$6x + 8 = 6x + 9 - 1$$

$$6x + 8 = 6x + 8$$

$$6x + 8 = 6x + 8$$

Is 17 a solution?

Yes

Is -42 a solution? Yes.

Ex $4(x-7) + 39 = 3(x+4) + x$

$$4x - 28 + 39 = 3x + 12 + x$$

$$4x + 11 = 4x + 12$$

$$4x + 11 - 4x = 4x + 12 - 4x$$

$$11 = 12$$

No solutions!

Is 17 a solution? No.

Is -42 a sol?

No.

Ex Emil had \$1000, deposited \$60 each month.

Jackie had \$2200, deposited \$60 each month.

How long until they have an equal bank balance?

how many months passed

$$1000 + 60m = 2200 + 60m$$

$$1000 + 60m - 60m = 2200 + 60m - 60m$$

$$1000 = 2200$$

FALSE \Rightarrow no solutions.