action that changes y; Goal $y = \#$
So undo the change!

1. Solve $y - 5 = -18$ ⟹ Attack "-5" by adding 5

$y - 5 + 5 = -18 + 5$

$y = -13$

---

2. Solve $z + 13 = -15$ (Action: add 13 to mystery number; undo!

$z + 13 - 13 = -15 - 13$

$z = -28$

---

3. Solve $-8 + y = -29$ (Action is adding -8 to the mystery number.

$-8 + y - (-8) = -29 - (-8)$

$-8 + y + 8 = -29 + 8$ ⟹ $y = -21$

---

4. Solve $x + \frac{7}{8} = \frac{9}{8}$

$x + \frac{7}{8} - \frac{7}{8} = \frac{9}{8} - \frac{7}{8}$ → $x = \frac{2}{8}$

$x = \frac{1}{4}$

---

5. Solve $20 - 7s = 26 - 8s$ (Goal: $s = \#$)

$20 - 7s + 8s = 26 - 8s + 8s$ — attack! (because ultimately don't want variables on the right!

add 20 to attack... $20 + s = 26$

Subtract 20 $20 + s - 20 = 26 - 20$

$s = 6$

---

6. Solve $7x + 3 = 6(x - 1) + 9$

<Simplify sides first>

$7x + 3 = 6x - 6 + 9$

$7x + 3 = 6x + 3$

$7x + 3 - 6x = 6x + 3 - 6x$ — variable on right... needs to be dealt with

$x + 3 = 3$

$x + 3 - 3 = 3 - 3$

$x = 0$

Ex Natia's bday. Tells you, 3 years ago, she was 20. How old is she now?

1) Identify what your are looking for and give it an algebra name

We'll let A represent her age.

2) Translate background into an equation

3 years ago, she was 20

back then, age was A − 3

"to be" verb becomes equals sign

$$A - 3 = 20$$

3) Solve:

$$A - 3 + 3 = 20 + 3$$

$$A = 23$$

Ex In 5 years, I will be ~~twice~~ as old as my cousin, who was born 10 years after I was. How old am I now?

1) Identify what we are looking, give it a name: A = my age now.

2) Set up equation:

.... will be .....

In 5 years... twice as old as ....

$$A+5 = 2\cdot(\quad)$$

my cousin ... in 5 years

$$A+5 = 2\left(\underbrace{A-10}_{\text{my cousin's age right now}} + 5\right)$$

my cousin in 5 years

3) Solve:

$$A+5 = 2(A-10+5)$$

$$A+5 = 2(A-5)$$

$$A+5 = 2A-10$$

clear variables from right side

$$A+5-2A = 2A-10-2A$$

$$-A+5 = -10$$

$$-A+5-5 = -10-5$$

$$-A = -15$$

$$\frac{-A}{-1} = \frac{-15}{-1} \Rightarrow A = 15$$

Equation	Action Present That You Must Attack	How to Attack
Ex		
$n-3=20$	Subtraction of 3	Add 3 to both sides
$a+4=30$	Addition of 4	Subtract 4 from both sides
$12=x-2$	Subtraction of 2	Addition of 2 to both sides
$5x=4x+7$	Addition by $4x$	Subtract $4x$

Equation	First Step	Second Step
Ex		
$7x+3=6x+5$	Subtract $6x$ from both sides	Subtract 3 from both sides
$2r+3r+7=6r-4$	Combine Like Terms	Subtract $6r$ from both sides
$-2(7-g)=g+3$	Distribute	Subtract g from both sides

Definition A linear equation in one-variable

is any equation you could write as

$$a \cdot x + b = c \qquad (\text{as long as } a \neq 0)$$

Is this equation linear?

$$-6 - 8t = 4$$

$$-6 + (-8t) = 4$$

$$\underbrace{-8}_{a}t + \underbrace{(-6)}_{b} = \underbrace{4}_{c}$$ Yes!

Is this linear?

$$x = 9$$

$$\underbrace{1}_{a} \cdot x + \underbrace{0}_{b} = \underbrace{9}_{c}$$ Yes!

Is this linear?

$$6x^2 - 8 = 6$$

No! "x^2" is not permitted.

Is this linear?

$$14 = 14$$

$$\underbrace{0}_{a \neq 0!} x + 14 = 14$$

No! Requires a variable with a nonzero number multiplied by it

Def Two equations are called equivalent equations if they have the same solutions.

Ex $x+7=17$ $\qquad$ $x-6=4$

Are these equivalent?

Strategy: Solve them both... if they have the same solutions, they're equivalent.

$x+7-7=17-7$

$x=10$

$x-6+6=4+6$

$x=10$

Same, so these are equivalent equations.

Ex Same question: $t+3=23$ $\qquad$ $t=-20$

$\Rightarrow t+3-3=23-3$

$t=20$

$t=-20$

Not equivalent!

Fact We can always solve a linear equation. (It's never too difficult.

The Method: original linear equation

produce an equivalent equation

_____ = _____

_____ = _____

eventually: $x = \#$

2.2

Ex Solve $\frac{1}{4}x = 10$

mystery number; action: multiplied by $\frac{1}{4}$

Goal is to isolate x

opposite action: multiply by 4

both valid

action: multiply by $\frac{1}{4}$

opposite action: divide by $\frac{1}{4}$

$$\frac{1}{4}x = 10$$

$$4 \cdot \frac{1}{4}x = 4 \cdot 10$$

$$x = 40$$

$$\frac{1}{4}x = 10$$

$$\frac{\frac{1}{4}x}{\frac{1}{4}} = \frac{10}{\frac{1}{4}}$$

$$\frac{1}{4}x \div \frac{1}{4} = 10 \div \frac{1}{4}$$

$$\frac{1}{4}x \cdot \frac{4}{1} = 10 \cdot \frac{4}{1}$$

$$x = \frac{40}{1}$$

$$x = 40$$

Ex $5x = 20$ — multiply by 5 (opposite action)

divide by 5:

$$\frac{5x}{5} = \frac{20}{5}$$

$$x = 4$$

OR

multiply by $\frac{1}{5}$:

$$5x = 20$$

$$\frac{1}{5} \cdot 5x = \frac{1}{5} \cdot 20$$

$$x = 4$$

Ex $3x = -6$

* we will divide by 3, because that is the opposite of mult. by 3

$$\frac{3x}{3} = \frac{-6}{3}$$

$$x = -2$$

Ex $-6x = 3$ — multiplication by -6; attack by dividing by -6

$$\frac{-6x}{-6} = \frac{3}{-6}$$

$$x = -\frac{1}{2}$$

Ex Solve $-x = 10$

What action is here? multiply by -1. the opposite is... divide by -1.

$$\frac{-x}{-1} = \frac{10}{-1}$$

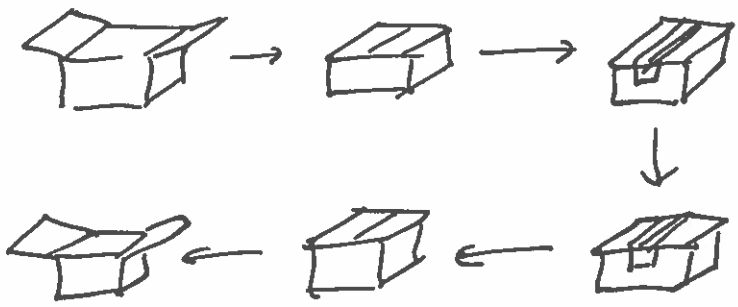
$$x = -10$$

One-Step

More-than-one Step

Equation	First Step	Second Step	Do It
$2x+8=12$	Subtract 8 from both sides	Divide by 2 on both sides.	$2x+8 = 12$
↑			$2x+8-8 = 12-8$
x...			$2x = 4$
multiplied by 2...			$\frac{2x}{2} = \frac{4}{2}$
added by 8...			$x = 2$

UNDO THESE!



So last action taken is first action to undo.

Ex $3x - 5 = 10$ Solve for x.

Last action: -5
First thing to attack: $+5$

$$3x - 5 + 5 = 10 + 5$$
$$3x = 15$$
$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5$$

Check!

$3(5) - 5 \stackrel{?}{=} 10$
$15 - 5 \stackrel{?}{=} 10$
$10 = 10$ ✓

Ex Solve $y + 2 = 2y + 9$

Attack because I don't want variables on the right!

$$y + 2 - 2y = 2y + 9 - 2y$$
$$-y + 2 = 9$$

Last action: "$+2$"
First thing to attack: "-2"

$$-y + 2 - 2 = 9 - 2$$
$$-y = 7$$
$$\frac{-y}{-1} = \frac{7}{-1}$$

$y = -7$

$-7 + 2 \stackrel{?}{=} 2(-7) + 9$
$-5 \stackrel{?}{=} -14 + 9$
✓

Solve $2a+3 = 5(a-2)+3a$

$2a+3 = 5a-10+3a$

$2a+3 = 8a-10$

$2a+3-8a = 8a-10-8a$

$-6a+3 = -10$

$-6a+3-3 = -10-3$

$-6a = -13$

$\frac{-6a}{-6} = \frac{-13}{-6}$

$a = \frac{13}{6}$

First step...
Not in simple
$ax+b=c$
yet!
Try to simplify...
→ Distributive Law.
→ Combine Like Terms
→ Get "a" off
the right side--
→ Undo last action
(+3)
→ Last step...

Try to check!

$2\left(\frac{13}{6}\right)+3 \stackrel{?}{=} 5\left(\frac{13}{6}-2\right)+3\left(\frac{13}{6}\right)$

$\frac{2}{1}\cdot\frac{13}{6_3}+3 \stackrel{?}{=} 5\left(\frac{13}{6}-\frac{12}{6}\right)+\frac{3}{1}\cdot\frac{13}{6_2}$

$\frac{13}{3}+3 \stackrel{?}{=} 5\cdot\frac{1}{6}+\frac{13}{2}$

$\frac{13}{3}+\frac{9}{3} \stackrel{?}{=} \frac{5}{6}+\frac{13\cdot 3}{2\cdot 3}$

$\frac{22}{3} \stackrel{?}{=} \frac{5}{6}+\frac{39}{6}$

$\frac{22}{3} \stackrel{?}{=} \frac{44}{6}$

✓

Ex the weight W (in pounds) of a plastic tank holding G gallons of water can be modeled:

$$W = 8.354G + 67$$] a formula.

A truck hauling this tank has max capacity 2000 lb.

we have a W...

How much volume of water is it safe to transport?

Looking for a G.

$$2000 = 8.354G + 67$$

(A linear equation... can be solved)

Attack last action first

$$2000 - 67 = 8.354G + 67 - 67$$

$$1933 = 8.354G$$

$$\frac{1933}{8.354} = \frac{8.354G}{8.354}$$

$$231.39\ldots = G$$

This truck can haul $\underset{\text{approximately}}{231.4}$ gallons of water.

1. Solve $\frac{x}{7} = 4$

$\frac{x}{7} \cdot 7 = 4 \cdot 7$

$x = 28$

---

2. Solve $\frac{x}{-5} = 8$

$\frac{x}{-5}(-5) = 8(-5)$

$x = -40$

---

3. Solve $-54 = -9z$

$\frac{-54}{-9} = \frac{-9z}{-9}$

$6 = z$ OR $z = 6$

---

4. Solve $-8x = 4$

$\frac{-8x}{-8} = \frac{4}{-8}$

$x = -\frac{1}{2}$

---

5. Solve $-x = 23$

$\frac{-x}{-1} = \frac{23}{-1}$

$x = -23$

---

6. Solve $-51 = -y$

$\frac{-51}{-1} = \frac{-y}{-1}$

$51 = y$ OR $y = 51$

---

7. Solve $9x + 2 = 6x - 4$

$9x + 2 - 6x = 6x - 4 - 6x$

$3x + 2 = -4$

$3x + 2 - 2 = -4 - 2$

$3x = -6$

$\frac{3x}{3} = \frac{-6}{3}$

$x = -2$

---

8. Solve $-3y - 2 = -5 - 4y$

$-3y - 2 + 4y = -5 - 4y + 4y$

$y - 2 = -5$

$y - 2 + 2 = -5 + 2$

$y = -3$

---

9. Solve: "Negative five times a number, increased by 11, is -29."

$-5x + 11 = -29$

$-5x + 11 - 11 = -29 - 11$

$-5x = -40$

$\frac{-5x}{-5} = \frac{-40}{5}$

$x = 8$