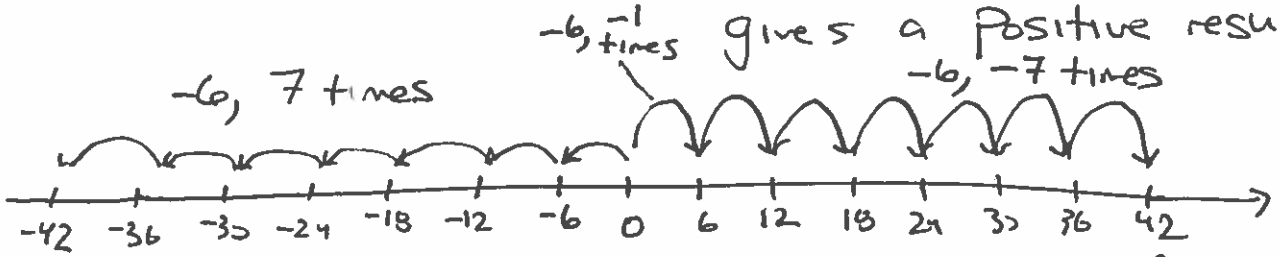
1.7 Multiplying & Dividing with Negative Numbers

Ex $(-6)(-7) = 42$

FACT:
Neg # times Neg # gives a Positive result



$(-6)(7) = -42$

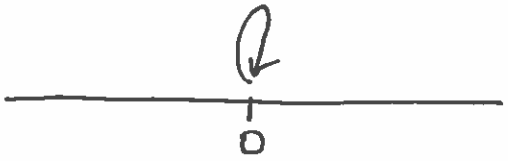
$(-6)(-7) = 42$

times table 6×8

Ex: $(-8) \cdot 6 = -48$

FACT: neg # × pos # = neg result.

Ex $0 \cdot (-2) = 0$



FACT: 0 times any number gives 0.

Ex $\dfrac{x^2 + 8x - 17}{[3-x]^2 + 28} \cdot 0 = 0$

Ex $\frac{1}{3}(-6)$

pos · neg = neg

$\frac{1}{3} \times 6 = 2$

$\frac{1}{3} \cdot \frac{6}{1} = \frac{6}{3} = 2$

So: $\frac{1}{3}(-6) = -2$

Ex $-\frac{3}{4}(-8) = ? = 6$

neg × neg = pos

$\frac{3}{4} \cdot 8$

$= \frac{3}{1\cancel{4}} \cdot \frac{\cancel{8}^{2}}{1}$

$= \frac{6}{1} = 6$

Ex $(-4)(-3)(-2) = ? = -24$

Short

neg × neg × neg

pos × neg

neg

$4 \cdot 3 \cdot 2$

$= 12 \cdot 2$

$= 24$

$\begin{array}{r} {}^{1}16 \\ \times\ 3 \\ \hline 48 \end{array}$

Ex $(-4)(-4)(3)(-2)(-1)(-4) = ?$

two here, two here, one

neg, result

$4 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 4$

$= 16 \cdot 3 \cdot 2 \cdot 1 \cdot 4$

$= 48 \cdot 2 \cdot 1 \cdot 4$

$= 96 \cdot 1 \cdot 4$

$= 96 \cdot 4$

$\begin{array}{r} {}^{3}96 \\ \times\ 4 \\ \hline 384 \end{array}$

So... $= -384$

Ex $-5(4x) = -20x$

neg × view as positive ⟹ neg sign part of result

$5 \cdot (4x) = (5 \cdot 4)x$
$= 20x$

Alternate

$-5(4x)$
$= (-5 \cdot 4)x$
$= -20x$

Ex $2(-3x) = -6x$

negative, not subtraction
(to its left, no number)

Multiply a pos times a neg...
So result has one minus sign (negative)

Ex $-6(-9x) = 54x$

two neg signs
→ positive

Distributive Property: $a \cdot (b + c) = a \cdot b + a \cdot c$

Extended version: $a \cdot (b - c) = a \cdot b - a \cdot c$

$= a(b + (-c))$

$= a \cdot b + a(-c)$

$= a \cdot b + a(-1 \cdot c)$

$= a \cdot b + (a \cdot -1) \cdot c$

$= a \cdot b + (-1 \cdot a)c$

$= a \cdot b + -1(a \cdot c)$

$= a \cdot b + -(ac)$

$= a \cdot b - ac$

Ex $2(3x - 4) = ? = 2(3x) - 2(4)$

$= 6x - 8$

addition or subtraction ✓

Ex $-3(x + 5) = -3 \cdot x + (-3)(5)$

$= -3x + (-15)$

$= -3x - 15$

Ex $-2(5x - 1) = -2(5x) - (-2)(1)$

$= -10x - (-2)$

$= -10x + 2$

subtraction or add ✓

Ex: $4(-3x + 5) = -12x + 20$

Ex: $6(-3x - 2) = -18x - 12$

Ex $-5(-3x - 1) = 15x + 5$

Sometimes you subtract a whole group of numbers.

this subtraction applies to 2x and 3.

Ex $(6x+5) - (2x+3)$

$= (6x+5) - 2x - 3$

Subtract all the parts, if the whole group was subtracted.

$= 6x + 5 - 2x - 3$

$= 4x + 2$

Ex $(4x+7) - (5x-9) = (4x+7) - (5x + (-9))$

$= 4x + 7 - 5x - (-9)$

$= 4x + 7 - 5x + 9$

$= -x + 16$

Ex $(5x-1)-(2x+6)$

$= 5x - 1 - 2x - 6$

$= 3x \quad ?$

$= 5x + (-1) + (-2x) + (-6)$

$= 3x + (-7)$

$= 3x - 7$

Division: $a \div b$ or $\frac{a}{b}$

Pos/neg rules are same for division as for multiplication:

pos ÷ pos = pos

pos ÷ neg = neg

neg ÷ pos = neg

neg ÷ neg = pos

Ex $(-15) \div 5 = -3$

neg ÷ pos

neg result

$15 \div 5 = 3$

Ex $16 \div (-4) = -4$

neg result

since one pos, one neg.

$16 \div 4 = 4$

Ex $\frac{-6}{3} = -2$

$\frac{\text{neg}}{\text{pos}} = \text{neg result}$

$\frac{6}{3} = 2$

Ex $\frac{-20}{-5} = 4$

fraction bar ⟹ division!

neg # div neg #

⟶ pos result

$\frac{20}{5} = 4$

Ex $0 \div (-2) = 0$

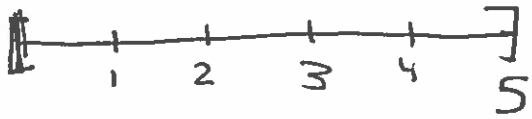
anything other than 0

FACT $0 \div$ (anything) $= 0$

Ex $5 \div 0 =$ no numerical result

$5 \div 0$ is undefined

Asking you to divide 0 into 5



Fact:

(Anything) $\div 0$

is undefined

Ex $\frac{0}{8} = 0$

(0 ÷ anything gives 0)

Ex $\frac{8}{0}$ is undefined

Dividing with fractions

Ex $2 \div \left(-\frac{3}{4}\right) = -\frac{8}{3}$

pos ÷ neg → neg result

$$2 \div \frac{3}{4}$$
$$= 2 \cdot \frac{4}{3}$$
$$= \frac{2}{1} \cdot \frac{4}{3}$$
$$= \frac{8}{3}$$

Dividing by a fraction is <u>same</u> as multiplying by its <u>reciprocal</u>

Ex $\left(-\frac{2}{3}\right) \div \left(\frac{3}{4}\right) = -\frac{8}{9}$

negative result ✓

$$\frac{2}{3} \div \frac{3}{4}$$
$$= \frac{2}{3} \cdot \frac{4}{3}$$
$$= \frac{8}{9}$$

Ex $\left(-\frac{2}{3}\right) \div \left(-\frac{6}{7}\right) = \frac{7}{9}$

neg ÷ neg gives pos

$$\frac{2}{3} \div \frac{6}{7}$$
$$= \frac{\cancel{2}^{1}}{3} \cdot \frac{7}{\cancel{6}_{3}}$$
$$= \frac{7}{9}$$

← anything to simplify? Don't try! (division, not multiplication)

Now could try to simplify

1.8 Exponents & Order of operations

bet $5 with double-or-nothing rules.

win, bet it all,
repeat until you won 4 times...
how much do you take?

$$\$5 \cdot \overbrace{2 \cdot 2 \cdot 2 \cdot 2}^{4 \text{ times}} = 5 \cdot 2^4$$

exponent

power

base

Ex $7^2 = \overbrace{7 \cdot 7}^{2 \text{ times}}$

$= 49$

Ex $(-3)^2 = \overbrace{(-3) \cdot (-3)}^{2 \text{ times}}$

$= 9$

Ex $(-2)^5 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2)$

$= -32$

Ex -2^5

negated

raised to 5th power

which happens first?

Order of Operations

high priority

P E M D A S

low priority

P: grouping symbols, parentheses

E: exponents

M D: multiplication division

A S: addition subtraction

Ex -5^2 Exponent first, it applies to 5.

$= -25$ and $5^2 = 25$

Ex $(-5)^2$ Parentheses has highest priority, multiplication by -1, then do the exponent

$= (-5)(-5)$

$= 25$

Ex $3 \cdot 4^2$ (E)

$= 3 \cdot 16$ (M)

$= 48$

Ex $(3 \cdot 4)^2$ (P, M)

$= 12^2$ (E)

$= 144$

Ex $2 + 3 \times 4$ (M has higher priority)

$= 2 + 12$ (A)

$= 14$

Ex $-5 + 20 \div 4$ (D)

$= -5 + 5$

$= 0$ (A)

Ex $(-2)^3 + 5 \cdot 4 \div 2$ (E, A, M, D)

$= -8 + 5 \cdot 4 \div 2$ (A, M, D)

$= -8 + 20 \div 2$

$= -8 + 10 = 2$

Side: $(-2)^3$

$= (-2)(-2)(-2)$

$= -8$

[With ties, prioritize left to right]

Ex $(10+3)^2 + 8 \div 2 \cdot 2$

P, E, A, D, M

$= 13^2 + 8 \div 2 \cdot 2$

E

$= 169 + 8 \div 2 \cdot 2$

A, D, M

$= 169 + 4 \cdot 2$

$= 169 + 8$

$= 177$

PE(MD)AS

M & D tie for priority break the tie by reading left to right.

$169 + 8 \div 2 \cdot 2$

wrongly done · first...

$= 169 + 8 \div 4$

$= 169 + 2$

$= 171$

Ex $\dfrac{15 + 20 \div (-4)}{3 - (-2)}$ ← D, but also makes two groups!

S

$= \dfrac{10}{5}$ ← top group ← before ← bottom group

$= 2$

one group: $15 + 20 \div (-4)$ (D)

$= 15 + (-5)$

$= 10$

2nd group: $3 - (-2)$ (S)

$= 5$

#152 Practice Test

$-2^2 - 5[10 - (7 - 4^3)]$ (E within a P within a P)

$= -2^2 - 5[10 - (7 - 64)]$ (S)

$= -2^2 - 5[10 - (-57)]$ (S)

$= -2^2 - 5[10 + 57]$ (A)

$= -2^2 - 5 \cdot 67$ (E)

$= -4 - 5 \cdot 67$ (M)

$= -4 - 335$

$= -339$

A represents your age

$$\frac{\left[(A + 10) \cdot 4 - 20\right] \cdot \frac{1}{2} - 10}{2}$$

A, but not like terms!

(M)

Simplify:

$$= \frac{\left[4 \cdot A + 40 - 20\right] \cdot \frac{1}{2} - 10}{2}$$

A, but not like terms

(S)

$$= \frac{\left[4A + 20\right] \cdot \frac{1}{2} - 10}{2}$$

2

(M)

$$= \frac{2A + 10 - 10}{2}$$

S

$$= \frac{2A}{2}$$

$$= \frac{A}{1} = A$$