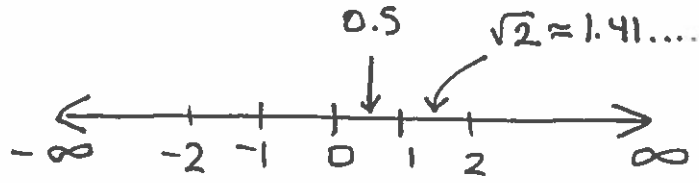
1.3 Real Numbers



Special Collections of Numbers

set notation; braces indicate a collection

Natural Numbers $\mathbb{N}$: $\{1, 2, 3, \ldots\}$

Whole Numbers $\mathbb{W}$: $\{0, 1, 2, 3, \ldots\}$

Integers $\mathbb{Z}$: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational Numbers: ratios of integers

look like $\frac{m}{n}$ (m, n integers, $n \neq 0$)

Ex $\frac{1}{2}$, $\frac{10}{17}$, $\frac{0}{5}$, $\frac{-3}{13}$, $\frac{-13}{3}$, ...

$\mathbb{Q}$ (fractions, decimals that terminate, decimals that repeat)

Ex $\frac{1}{2}$, 2.36, 0.333..., but not 3.1415926...

Irrational Numbers: Not rational...

in decimal form, the decimal goes on forever without repeating

$\pi = 3.1415926\ldots$

$\sqrt{2} = 1.4142\ldots$

Ex 2.36 ⟶ 2 and 36 hundredths

$2 + \frac{36}{100} = \frac{200}{100} + \frac{36}{100}$

$= \frac{236}{100}$

↓ terminating decimal → rational number (means $\frac{m}{n}$)

Ex $0.12121212\ldots$

aka $0.\overline{12}$ — how is this a fraction

$0.12\,12\,12\,12\ldots$

repetition starts right away

$= \frac{12}{99}$ ← two digits 9, because repeated part had two digits.

reduce

$= \frac{4}{33}$

Ex $-10,\ -5.\overline{3},\ 0,\ 2.6,\ \pi,\ \sqrt{15},\ \sqrt{16}$ (need to recognize this is 4)

Natural: $\sqrt{16}$

Whole: $0,\ \sqrt{16}$

Integers: $-10,\ 0,\ \sqrt{16}$

Rational: $-10,\ -5.\overline{3},\ 0,\ 2.6,\ \sqrt{16}$

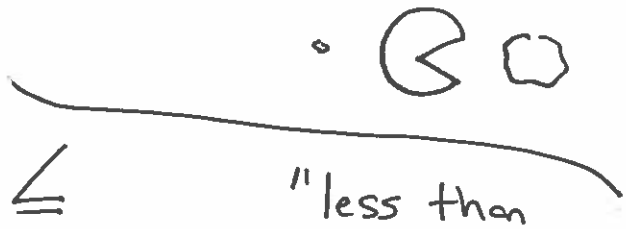
Irrational: $\pi,\ \sqrt{15}$

Inequalities

Symbols

$<$ "less than"

small $<$] large



$\leq$ "less than or equal to"

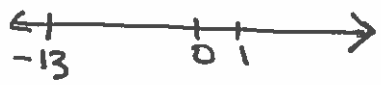
$5 \leq 7.1$

$5 \leq 5$

Examples

$5 < 7.1$

$-13 < 1$



$-13 < -12$

daughter's age $<$ mother's age

cost of admission to a movie $\leq$ how much money I have

And $>$, $\geq$

make both fractions 0.666...

Ex $|-5| < 7$ $\quad |-13| > |-2|$ $\quad \frac{2}{3} > 0.6$

$|-5|$ means this distance, 5

1.4 Basic Rules of Algebra

Math expressions like $7x + 3$

called terms

components of an expression that are added together.

Like terms:
terms with same variable factor

$7x$

called factors

components that are multiplied together

$7x$

coefficient

variable factor

Ex

Like terms	Unlike terms
$2x$, $3x$	$7x$, 2
x^2, $\frac{1}{5}x^2$	$15x$, $15y$
$3.7y$, πy	$7x^2$, $2x$
$-13x^2y$, $31x^2y$	

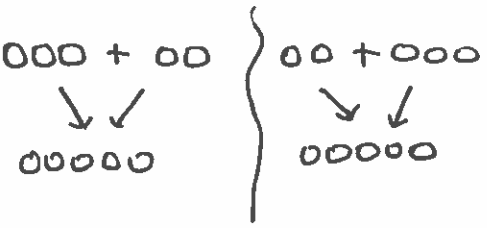
Ex $3x + 18 + 5.1x + 2x + 12$ (five terms)

like terms

like terms

Fact: $3 + 2 = 2 + 3$

$a + b = b + a$



Commutative property of addition

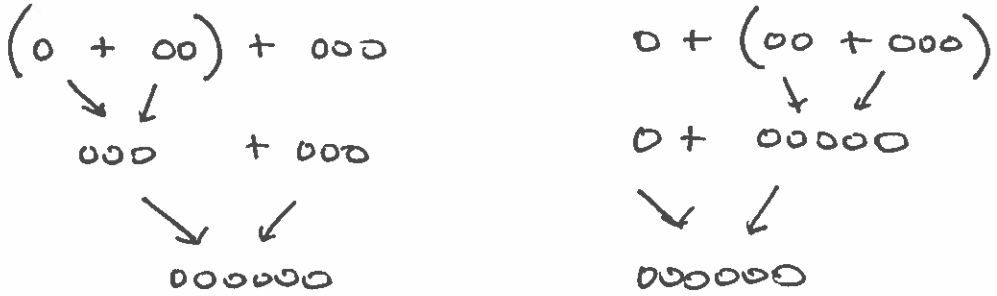
(order of addition doesn't matter)

$a + b = b + a$

Fact: $3 \cdot 2 = 2 \cdot 3$

commutative property of multiplication

Fact $(1 + 2) + 3 = 1 + (2 + 3)$



$(a + b) + c = a + (b + c)$

Associative property of addition (moving parentheses)

Fact: $(a \cdot b) \cdot c = a(b \cdot c)$

Associative property of multiplication

Distributive Property $a \cdot (b+c) = a \cdot b + a \cdot c$

$2(1+3)$ $\quad$ $2(1) + 2(3)$

$2(4)$ $\quad$ $2 + 6$

8 $\quad$ 8

Ex Simplify $7 + x + 2 + 5x$

↑ first addition

$$((7+x)+2)+5x$$

$$=((x+7)+2)+5x \quad \leftarrow \text{comm. prop of addition}$$

$$=(x+(7+2))+5x \quad \leftarrow \text{assoc. prop of addition}$$

$$=(x+9)+5x$$

$$=(9+x)+5x$$

$$=9+(x+5x)$$

$$=9+6x$$

$$=6x+9$$

When adding terms together, its ok (no subtraction) to just add like terms.

$7 + x + 2 + 5x$

$9 + 6x$

Simplify:

Ex $3 + (2x+1)$

$= 4 + 2x$

Ex $(3x+5) + (5x+9)$

all about addition of terms.

$= 8x + 14$

Simplify

Ex $2(5x)$

$= 10x$

Simplify

Ex $3(2xy)$

$= (3 \cdot 2)xy$

$= 6xy$

Ex $(7x)(6)$

$= 42x$

Ex ~~Simplify~~

Expand (using distribute property)

$2(x+4)$

$a(b+c)$

$= a \cdot b + a \cdot c$

$= 2x + 8$

Ex $3(2x^2 + 5)$

$= \square(\square + \square)$

$= 3 \cdot 2x^2 + 3 \cdot 5$

$= 6x^2 + 15$

Simplify:

Ex $2(3x+4) + 5x$

$= 2 \cdot 3x + 2 \cdot 4 + 5x$

$= 6x + 8 + 5x$

$= 11x + 8$

1. Use the distributive property: $4(x+6) = 4x + 24$

2. Use the distributive property: $9(2x+5) = 18x + 45$

3. Use the distributive property: $6(3x-2) = 18x - 12$

4. Use the distributive property: $\frac{1}{3}(7x - 21)$

$= \frac{1}{3} \cdot 7x - \frac{1}{3} \cdot 21$

$= \frac{1}{3} \cdot \frac{7}{1} x - \frac{1}{3} \cdot \frac{21}{1}$

$= \frac{7}{3}x - 7$

$\frac{1}{3} \cdot \frac{21}{1}$ 7

5. Simplify $5x + 13x = 18x$

6. Simplify $14b - 5b = 9b$

7. Simplify $13a + 15 + 2a + 11 = 15a + 26$

8. Simplify $2(5x+4) - 3 = 10x + 8 - 3$

$= 10x + 5$

9. Name the property used in each step

$7 + 2(x+9)$

$= 7 + (2x + 18)$ Distributive

$= 7 + (18 + 2x)$ Comm. Prop of Add.

$= (7 + 18) + 2x$ Assoc. Prop of Add

$= 25 + 2x$ Added..

$= 2x + 25$ Comm. Prop of Add.

1.5 Working with Negative Numbers (Addition)

Ex $-20 + 20$

$= 0$

(a number and its negative add to 0.)

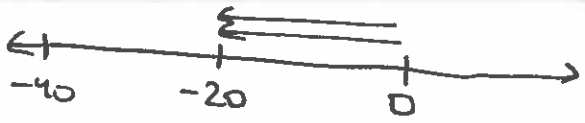
Ex $20 + (-20)$

$= 0$

adding a neg #, can use parens.

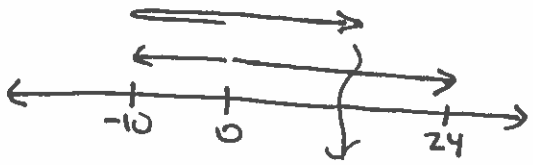
Ex $-20 + (-20)$

$= -40$



Ex $-10 + 24$

$= 14$

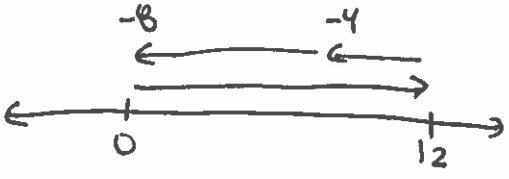


recognize it's a positive result.

Ex $12 + (-4) + (-8)$

$= 8 + (-8)$

$= 0$



Ex $\frac{-3}{8} + \left(\frac{-1}{8}\right)$

$= -\frac{4}{8} = \frac{-1}{2}$

OK, fractions... still think through is the result pos or neg ✓

Ex $\frac{-3}{4} + \frac{5}{4}$ (will result be pos or neg?)

$= \frac{2}{4}$ ← two numbers are in opposite directions... common denominator... difference between 5 & 3.

$= \frac{1}{2}$

Ex $-\frac{1}{2} + \left(-\frac{3}{4}\right)$ (Same question: neg result)

← make common denominator

$= -\frac{2}{4} + \left(\frac{-3}{4}\right)$

$= -\frac{5}{4}$ ← (two numbers in same direction Combine 2 & 3)

Ex Simplify $-12x + 15x$. Like terms ✓

$= 3x$

Ex $-6x + (-4x)$ Like terms ✓

$= -10x$

Ex $-6x^2 + (-4x)$ Not like terms. Don't do anything.

1. Calculate $6 + (-14)$
2. Calculate $8 + (-5)$
3. Calculate $-8 + (-5) + 10$
4. Calculate $-12 + 5 + 4 + (-2)$
5. Simplify $-\frac{9}{10} + \frac{1}{4}$
6. Simplify $-\frac{1}{5} + \frac{3}{10}$
7. Simplify $-\frac{2}{3} + \frac{1}{4}$
8. Simplify $8x + (-9x)$
9. Simplify $-2x + x$
10. Simplify $-2x + 5x + (-6x)$
11. Simplify $-6x + 10 + 8x + (-12)$
12. Simplify $5x + (-7) + (-8x) + (-2)$