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MTH 261
Linear Algebra
Summer 2017
Determinant Computation

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Find the determinant of $\begin{bmatrix} 8 & -9 \\ 2 & -4 \end{bmatrix}$ using the special 2×2 determinant formula.

$$\det A = 8(-4) - (-9)(2)$$
$$= -32 + 18$$
$$= -14$$

2. Use the special 3×3 determinant rule to find $\begin{vmatrix} 2 & 1 & -5 \\ 2 & 2 & -4 \\ 1 & 3 & 1 \end{vmatrix}$.

$$\det A = 2\cdot 2\cdot 1 + 1(-4)(1) + (-5)(2)(3)$$
$$- 2(-4)(3) - 1(2)(1) - (-5)(2)(1)$$
$$= 4 - 4 - 30 + 24 - 2 + 10$$
$$= 0 - 6 + 8$$
$$= 2$$

3. Use the (hyper)-volume definition of the determinant function to find the determinant of $\begin{bmatrix} 2 & 4 & 6 & 8 \\ 1 & 2 & -3 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & 2 \end{bmatrix}$

these rows are parallel.
So the 4D parallelepiped is "degenerate" and has 0 hypervolume.

4. Use row reduction to find $\begin{vmatrix}1&-2&1&0\\0&2&4&8\\0&2&4&-3\\1&-2&-2&-9\end{vmatrix}$.

$$\xrightarrow{\text{replacement}} \begin{bmatrix}1&-2&1&0\\0&2&4&8\\0&2&4&-3\\0&0&-3&-9\end{bmatrix}$$

$$\xrightarrow{\text{replacement}} \begin{bmatrix}1&-2&1&0\\0&2&4&8\\0&0&0&-11\\0&0&-3&-9\end{bmatrix} \xrightarrow{\text{swap}} \underbrace{\begin{bmatrix}1&-2&1&0\\0&2&4&8\\0&0&-3&-9\\0&0&0&-11\end{bmatrix}}_{\text{has det } 1\cdot 2(-3)(-11)=66}$$

original det is -66.

5. Find the determinant of $\begin{bmatrix}2&1&1&1\\2&-1&0&1\\1&0&0&4\\0&1&-2&3\end{bmatrix}$ by expanding across a row~~s~~ or column~~s~~.

$$1\cdot\det\begin{bmatrix}1&1&1\\-1&0&1\\1&-2&3\end{bmatrix} - 4\cdot\det\begin{bmatrix}2&1&1\\2&-1&0\\0&1&-2\end{bmatrix} = -\det\begin{bmatrix}-1&1\\1&3\end{bmatrix} - (-2)\det\begin{bmatrix}1&1\\-1&1\end{bmatrix}$$

$$-4\left(-\det\begin{bmatrix}2&1\\2&0\end{bmatrix} + (-2)\det\begin{bmatrix}2&1\\2&-1\end{bmatrix}\right)$$

$$= -(-3-1) + 2(1+1) - 4\left(-(0-2) - 2(-2-2)\right)$$

$$= -(-4) + 4 - 4(2+8) = 8 - 40 = -32$$

6. Find the determinant of $\begin{bmatrix}2&-3&0&0\\1&0&4&5\\0&-2&-2&0\\-1&0&0&4\end{bmatrix}$ using the permutations-based definition of the determinant.

odd odd even

$$-(2)(4)(-2)(4) - (-3)(1)(-2)(4) + (-3)(5)(-2)(-1)$$

$$= 64 - 24 - 30 = 10$$