Date ____________

MTH 261
Linear Algebra
Spring 2017
Vector Equations

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Write a vector equation that is equivalent to the system below. There's no requirement to solve the system.

$$x_2 + 5x_3 = 0$$
$$4x_1 + 6x_2 - x_3 = 0$$
$$-x_1 + 3x_2 - 8x_3 = 0$$

$$x_1\begin{bmatrix}0\\4\\-1\end{bmatrix} + x_2\begin{bmatrix}1\\6\\3\end{bmatrix} + x_3\begin{bmatrix}5\\-1\\-8\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

2. Determine if $\begin{bmatrix}2\\-1\\6\end{bmatrix}$ is a linear combination of $\begin{bmatrix}1\\-2\\0\end{bmatrix}$ and $\begin{bmatrix}1\\1\\2\end{bmatrix}$.

$$\begin{bmatrix}1 & 1 & 2\\-2 & 1 & -1\\0 & 2 & 6\end{bmatrix} \xrightarrow{RREF} \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

inconsistent!

No!

3. Determine if $\begin{bmatrix}2\\-1\\6\end{bmatrix}$ is a linear combination of $\begin{bmatrix}1\\-2\\0\end{bmatrix}$, $\begin{bmatrix}1\\1\\2\end{bmatrix}$ and $\begin{bmatrix}2\\-7\\-2\end{bmatrix}$.

$$\begin{bmatrix}1 & 1 & 2 & 2\\-2 & 1 & -7 & -1\\0 & 2 & -2 & 6\end{bmatrix} \xrightarrow{RREF} \begin{bmatrix}1 & 0 & 3 & 0\\0 & 1 & -1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

inconsistent!

No!

4. Determine if $\begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ -2 & 1 & 0 & -1 \\ 0 & 2 & -2 & 6 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \end{bmatrix}$$

So, yes! It <u>is</u>.

5. Let $\vec{a}_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$, $\vec{a}_2 = \begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} 3 \\ -5 \\ h \end{bmatrix}$. For what values of h is $\vec{b}$ a linear combination of $\vec{a}_1$ and $\vec{a}_2$?

$$\left[\begin{array}{cc|c} 1 & -5 & 3 \\ 3 & -8 & -5 \\ -1 & 2 & h \end{array}\right] \xrightarrow[R_1 + R_3 \to R_3]{-3R1 + R_2 \to R_2} \begin{bmatrix} 1 & -5 & 3 \\ 0 & 7 & -14 \\ 0 & -3 & 3+h \end{bmatrix} \xrightarrow{\frac{1}{7}R_2 \to R_2}$$

$$\begin{bmatrix} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & -3 & 3+h \end{bmatrix} \xrightarrow{3R_2 + R_3 \to R_3} \begin{bmatrix} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & -3+h \end{bmatrix}$$

Need $h = 3$.

6. Let $\vec{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$. Show that every $\begin{bmatrix} h \\ k \end{bmatrix}$ is in Span$\{\vec{u}, \vec{v}\}$. (Hint: look into the associated augmented matrix.)

$$\begin{bmatrix} 1 & 2 & h \\ 3 & -1 & k \end{bmatrix} \longrightarrow \begin{bmatrix} \boxed{1} & 2 & h \\ 0 & \boxed{-7} & -3h+k \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & h \\ 0 & 1 & \frac{-3h+k}{-7} \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} \boxed{1} & 0 & -2\left(\frac{-3h+k}{-7}\right) + h \\ 0 & \boxed{1} & \frac{-3h+k}{-7} \end{bmatrix}$$

Could stop here!