Date ______________

MTH 261
Linear Algebra
Spring 2017

Systems of Linear Equations

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Solve the linear systems using an augmented matrix and row operations. Practice documenting exactly what row operations you use from one step to the next.

(a) $\begin{cases} x + 3y + z = 2 \\ -2x + 2y - 4z = -1 \\ -y + 3z = 1 \end{cases}$

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ -2 & 2 & -4 & -1 \\ 0 & -1 & 3 & 1 \end{bmatrix} \xrightarrow{2R_1+R_2 \to R_2} \begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & 8 & -2 & 3 \\ 0 & -1 & 3 & 1 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & -1 & 3 & 1 \\ 0 & 8 & -2 & 3 \end{bmatrix}$$

$$\xrightarrow{8R_2+R_3 \to R_3} \begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & -1 & 3 & 1 \\ 0 & 0 & 22 & 11 \end{bmatrix} \xrightarrow{\frac{1}{22}R_3 \to R_3} \begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & -1 & 3 & 1 \\ 0 & 0 & 1 & 1/2 \end{bmatrix} \xrightarrow[-3R_3+R_2 \to R_2]{-R_3+R_1 \to R_1} \begin{bmatrix} 1 & 3 & 0 & 3/2 \\ 0 & -1 & 0 & -1/2 \\ 0 & 0 & 1 & 1/2 \end{bmatrix}$$

$$\xrightarrow[-R_2 \to R_2]{3R_2+R_1 \to R_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{bmatrix} \Longrightarrow \begin{aligned} x &= 0 \\ y &= 1/2 \\ z &= 1/2 \end{aligned}$$

(b) $\begin{cases} 2x_1 - 2x_2 - x_3 = -3 \\ x_1 - 3x_2 + x_3 = -2 \\ x_1 - 2x_2 = 2 \end{cases}$

$$\begin{bmatrix} 2 & -2 & -1 & -3 \\ 1 & -3 & 1 & -2 \\ 1 & -2 & 0 & 2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -3 & 1 & -2 \\ 2 & -2 & -1 & -3 \\ 1 & -2 & 0 & 2 \end{bmatrix} \xrightarrow[-R_1+R_3 \to R_3]{-2R_1+R_2 \to R_2} \begin{bmatrix} 1 & -3 & 1 & -2 \\ 0 & 4 & -3 & 1 \\ 0 & 1 & -1 & 4 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & -3 & 1 & -2 \\ 0 & 1 & -1 & 4 \\ 0 & 4 & -3 & 1 \end{bmatrix} \xrightarrow{-4R_2+R_3 \to R_3} \begin{bmatrix} 1 & -3 & 1 & -2 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 1 & -15 \end{bmatrix} \xrightarrow[R_3+R_2 \to R_2]{-R_3+R_1 \to R_1} \begin{bmatrix} 1 & -3 & 0 & 13 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & -15 \end{bmatrix}$$

$$\xrightarrow{3R_2+R_1 \to R_1} \begin{bmatrix} 1 & 0 & 0 & -20 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & -15 \end{bmatrix} \Longrightarrow \begin{aligned} x_1 &= -20 \\ x_2 &= -11 \\ x_3 &= -15 \end{aligned}$$

(c) $\begin{cases} -2x_1 + x_2 = 2 \\ 3x_1 - x_2 + 2x_3 = 1 \end{cases}$

$$\begin{bmatrix} -2 & 1 & 0 & 2 \\ 3 & -1 & 2 & 1 \end{bmatrix} \xrightarrow{R_1 + R_2 \to R_2} \begin{bmatrix} -2 & 1 & 0 & 2 \\ 1 & 0 & 2 & 3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2}$$

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ -2 & 1 & 0 & 2 \end{bmatrix} \xrightarrow{2R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 4 & 8 \end{bmatrix} \Longrightarrow$$

x_3 is free

$x_2 = 8 - 4x_3$

$x_1 = 3 - 2x_3$

(d) $\begin{cases} 2x_1 + 2x_2 - x_3 - x_4 = -3 \\ -x_2 + 3x_4 = 2 \end{cases}$

$$\begin{bmatrix} 2 & 2 & -1 & -1 & -3 \\ 0 & -1 & 0 & 3 & 2 \end{bmatrix} \xrightarrow{2R_2 + R_1 \to R_1} \begin{bmatrix} 2 & 0 & -1 & 5 & 1 \\ 0 & -1 & 0 & 3 & 2 \end{bmatrix} \xrightarrow[-R_2 \to R_2]{\frac{1}{2}R_1 \to R_1}$$

$$\begin{bmatrix} 1 & 0 & -1/2 & 5/2 & 1/2 \\ 0 & 1 & 0 & -3 & -2 \end{bmatrix} \Longrightarrow$$

x_4 is free

x_3 is free

$x_2 = -2 + 3x_4$

$x_1 = 1/2 + \frac{1}{2}x_3 - 5/2\, x_4$

2. Give restrictions on a (like $a = 2$, $a \neq 0$, $a > 5$, etc.) so that this linear system is consistent. Use row operations as usual, but you will have to keep track of what happens with a symbolically.

$-x + 3y = a$

$2x - 6y = 3$

$$\begin{bmatrix} -1 & 3 & a \\ 2 & -6 & 3 \end{bmatrix} \xrightarrow{2R_1 + R_2 \to R_2} \begin{bmatrix} -1 & 3 & a \\ 0 & 0 & 2a+3 \end{bmatrix}$$

will be consistent if and only if $2a + 3 = 0$.

So only for $a = -3/2$.

3. Give restrictions on a, b, and c (like $a+b=c$, $b-2a \neq c$, etc.) so that this linear system is consistent. Use row operations as usual, but you will have to keep track of what happens with a, b, and c symbolically.

$$x - y + 2z = a$$
$$2x + y - z = b$$
$$4x + 2y + z = c$$

$$\begin{bmatrix} 1 & -1 & 2 & a \\ 2 & 1 & -1 & b \\ 4 & 2 & 1 & c \end{bmatrix} \xrightarrow[-4R_1+R_3 \to R_3]{-2R_1+R_2 \to R_2} \begin{bmatrix} 1 & -1 & 2 & a \\ 0 & 3 & -5 & -2a+b \\ 0 & 6 & -7 & -4a+c \end{bmatrix}$$

$$\xrightarrow{-2R_2+R_3 \to R_3} \begin{bmatrix} 1 & -1 & 2 & a \\ 0 & 3 & -5 & -2a+b \\ 0 & 0 & 3 & -2(-2a+b)-4a+c \end{bmatrix}$$

It doesn't matter what a, b, c are. The system will always be consistent.

4. Find all values for a that would make the system inconsistent.

$$x - y = 2$$
$$3x - 3y = a$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & -3 & a \end{bmatrix} \xrightarrow{-3R_1+R_2 \to R_2} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & -6+a \end{bmatrix}$$

The system is inconsistent if and only if $-6+a \neq 0$
$a \neq 6$.

5. Find all values for a that would make the system inconsistent.

$$2x - y = a$$
$$6x - 3y = a$$

$$\begin{bmatrix} 2 & -1 & a \\ 6 & -3 & a \end{bmatrix} \xrightarrow{-3R_1+R_2 \to R_2} \begin{bmatrix} 2 & -1 & a \\ 0 & 0 & -2a \end{bmatrix}$$

The system is inconsistent if and only if $-2a \neq 0$
$a \neq 0$

6. Find an equation of the form $y = ax^2 + bx + c$ for the parabola passing through the three points $(0, 0.25)$, $(1, -1.75)$, and $(-1, 4.25)$. (From your perspective, a, b, and c are the variables that you are trying to solve for. Set up a system of linear equations using the points the parabola passes through.)

Solved in class:

7. Find a cubic polynomial $ax^3 + bx^2 + cx + d$ whose graph passes through the points $(1, 5)$, $(2, 13)$, $(-1, 7)$, and $(-2, -7)$.

Solved in class.