Date ______

MTH 261
Linear Algebra
Spring 2017
The Matrix of a Linear Transformation

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Let T from $\mathbb{R}^2$ to $\mathbb{R}^3$ be such that

$$T : \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x+y \\ 2x+y \\ 3x-y \end{bmatrix}$$

(a) Find the matrix for T with respect to the standard coordinate vectors, A_T.

(b) Find $T\left(\begin{bmatrix} 2 \\ -2 \end{bmatrix}\right)$ in two ways: by directly using the rule for T and by using the matrix A_T.

$$T(\vec{e}_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$T(\vec{e}_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\Longrightarrow A_T = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & -1 \end{pmatrix}$$

$$T\left(\begin{bmatrix} 2 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 2+-2 \\ 2(2)+-2 \\ 3(2)-(-2) \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 8 \end{bmatrix}$$

$\checkmark$

$$A_T \cdot \begin{bmatrix} 2 \\ -2 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 2\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 8 \end{bmatrix}$$

2. Identify the space of 2×2 matrices with $\mathbb{R}^4$ by identifying each matrix in

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

with a coordinate vector from $\mathbb{R}^4$. Let T be the linear transformation from $M_{2\times 2}$ to $\mathbb{R}^2$ given by $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \mapsto \begin{bmatrix} a+d \\ b-c \end{bmatrix}$.

(a) Find A_T. (A_T is a matrix — do you understand what dimensions it should have?)

(b) Find $T\left(\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\right)$ in two ways: by directly using the rule for T and by using the matrix A_T.

$$T : \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \mapsto \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$T : \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T : \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$T : \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \mapsto \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

So $A_T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$

$T\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, directly

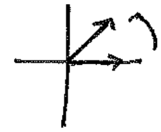
but

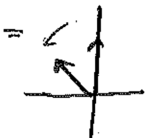
$$A_T \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the operator that rotates $\mathbb{R}^2$ about the origin counterclockwise by an angle of 45°.

   (a) Find the matrix for T with respect to the standard basis.

   (b) Find $T\left(\begin{bmatrix} 10 \\ 2 \end{bmatrix}\right)$.

$T(\vec{e}_1) =$  $= \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$

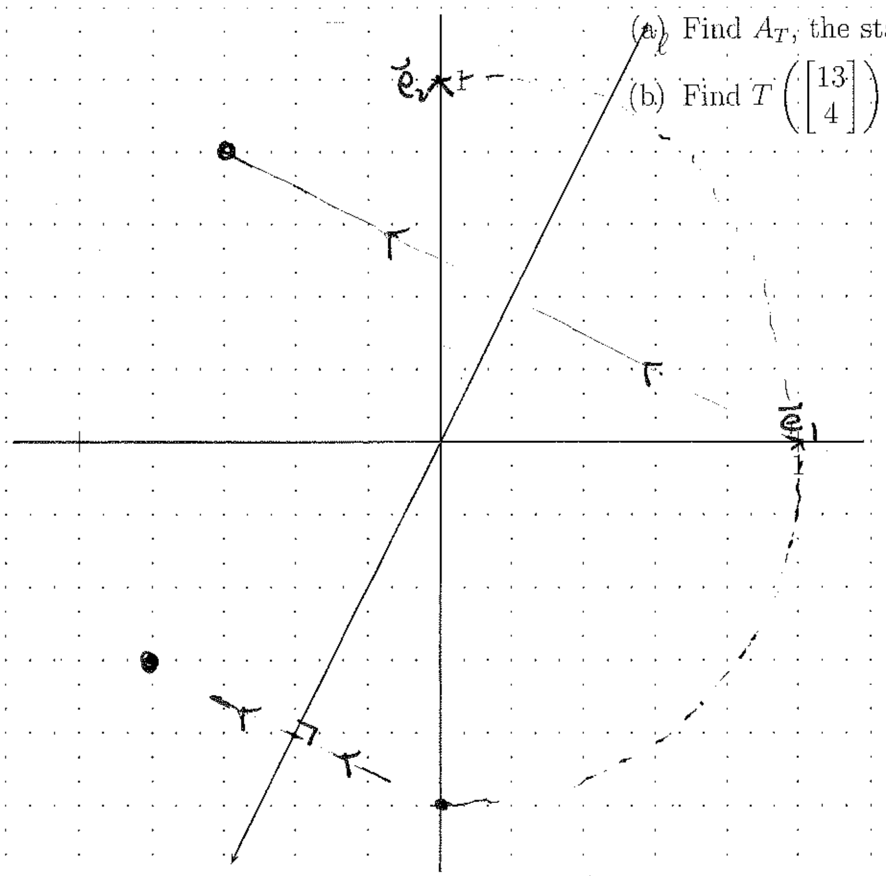
$T(\vec{e}_2) =$  $= \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$

$$A_T = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} = \frac{\sqrt{2}}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}\begin{bmatrix} 10 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4\sqrt{2} \\ 6\sqrt{2} \end{bmatrix}$$

4. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first rotates vectors about the origin by 90° clockwise, and then reflects perpendicularly through the line ℓ.



   (a) Find A_T, the standard matrix for T.

   (b) Find $T\left(\begin{bmatrix} 13 \\ 4 \end{bmatrix}\right)$.

$$T(\vec{e}_1) = \begin{bmatrix} -0.8 \\ -0.6 \end{bmatrix}$$

$$T(\vec{e}_2) = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix}$$

$$A_T = \begin{bmatrix} -0.8 & -0.6 \\ -0.6 & 0.8 \end{bmatrix}$$

So.. for example...
to rotate & reflect $\begin{bmatrix} -0.8 \\ 0.2 \end{bmatrix}$ ...

$$\begin{bmatrix} -0.8 & -0.6 \\ -0.6 & 0.8 \end{bmatrix}\begin{bmatrix} -0.8 \\ 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} .64 & -.12 \\ .48 & +.16 \end{bmatrix} = \begin{bmatrix} .52 \\ .64 \end{bmatrix}$$