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MTH 261
Linear Algebra
Spring 2017
Matrix Inverses

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Find the inverse of these matrices by using the special shortcut for 2×2 matrices.

$$A = \begin{bmatrix} 12 & 4 \\ 5 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 8 & 3 \\ -13 & -5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & -4 \\ -5 & 12 \end{bmatrix} \cdot \frac{1}{24-20}$$

$$= \begin{bmatrix} 1/2 & -1 \\ -5/4 & 3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{-40+39} \begin{bmatrix} -5 & -3 \\ 13 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 \\ -13 & -8 \end{bmatrix}$$

2. Suppose that A and AB are invertible. Can you conclude that B is invertible? Be careful how you answer this: at no point can you refer to B^{-1} until you have established that B is indeed invertible. You need to find a way to write something like $B \cdot (\text{something}) = \cdots = I$.

$\cancel{(B \cdot A)(BA)^{-1}}$ ~~why does this exist?~~

$$(AB)^{-1} \cdot A \cdot B = I$$

$$\left((AB)^{-1} \cdot A\right) \cdot B = I$$

So B has an inverse: $\left((AB)^{-1} \cdot A\right)$

3. Find the inverse of the following matrix using the row reduction method.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 3 & 9 & 9 & 0 & 1 & 0 \\ 0 & 4 & -5 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-3R_1+R_2\to R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & 3 & -3 & -3 & 1 & 0 \\ 0 & 4 & -5 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{3}R_2\to R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1/3 & 0 \\ 0 & 4 & -5 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-4R_2+R_3\to R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1/3 & 0 \\ 0 & 0 & -1 & 4 & -4/3 & 1 \end{array}\right] \xrightarrow{-R_3\to R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1/3 & 0 \\ 0 & 0 & 1 & -4 & 4/3 & -1 \end{array}\right]$$

$$\xrightarrow{R_3+R_2\to R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & 1 & 0 & -5 & 5/3 & -1 \\ 0 & 0 & 1 & -4 & 4/3 & -1 \end{array}\right] \xrightarrow{-4R_3+R_1\to R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 17 & -16/3 & 4 \\ 0 & 1 & 0 & -5 & 5/3 & -1 \\ 0 & 0 & 1 & -4 & 4/3 & -1 \end{array}\right]$$

$$\xrightarrow{-2R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 27 & -26/3 & 6 \\ 0 & 1 & 0 & -5 & 5/3 & -1 \\ 0 & 0 & 1 & -4 & 4/3 & -1 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} 27 & -26/3 & 6 \\ -5 & 5/3 & -1 \\ -4 & 4/3 & -1 \end{bmatrix}$$

4. Determine if each of the following matrices are invertible. There is no need to fully find their inverses (if they exist).

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -4 & 1 \\ -1 & 17 & -8 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & 1 \\ 2 & -1 & -3 \\ 1 & 3 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & -10 & 5 & -2 & 1 & 0 \\ 0 & 20 & -10 & 1 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 0 & 0 & 1 \\ 2 & -1 & -3 & 0 & 1 & 0 \\ 2 & 1 & 1 & 1 & 0 & 0 \end{array}\right] \quad \left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & -4 & -8 & -12 & -5 & 1 & 0 & 0 \\ 0 & -8 & -16 & -24 & -9 & 0 & 1 & 0 \\ 0 & -12 & -24 & -36 & -13 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & -10 & 5 & -2 & 1 & 0 \\ 0 & 0 & 0 & -3 & 2 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 0 & 0 & 1 \\ 0 & -7 & -5 & 0 & 1 & -2 \\ 0 & -5 & -1 & 1 & 0 & -2 \end{array}\right] \quad \left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & -4 & -8 & -12 & -5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -3 & 0 & 1 \end{array}\right]$$

Can't RREF to I. A is not invertible

will reduce to I, so B is invertible

Can't RREF to I. C is not invertible