Date ____________

MTH 261
Linear Algebra
Spring 2017

The Matrix Equation $A\vec{x} = \vec{b}$

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Compute the products all the way. Start by rewriting the product as a linear combination of vectors. Watch out — I may be trying to trick you.

$$\begin{bmatrix} -4 & 2 \\ 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix}$$

undefined !!!

$$\begin{bmatrix} 1 & 2 \\ -3 & 1 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = -2\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 6 \\ -2 \end{bmatrix} + \begin{bmatrix} 6 \\ 3 \\ 18 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 16 \end{bmatrix}$$

2. Without simplifying anything, rewrite the equation as a vector equation.

$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ -2 & -3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$

$$2\begin{bmatrix} 1 \\ -2 \end{bmatrix} + (-1)\begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \end{bmatrix} + (-1)\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$

3. Rewrite this vector equation as a matrix equation.

$$x_1 \begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$

3 variables b.c. matrix has 3 columns

4. Solve the equation $\begin{bmatrix} 1 & 3 & -4 \\ 1 & 5 & 2 \\ -3 & -7 & 6 \end{bmatrix} \vec{x} = \begin{bmatrix} -2 \\ 4 \\ 12 \end{bmatrix}$. If this equation doesn't quite makes sense to you, try rewriting it is a vector equation or as a system of linear equations.

$$\left[\begin{array}{ccc|c} 1 & 3 & -4 & -2 \\ 1 & 5 & 2 & 4 \\ -3 & -7 & 6 & 12 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$x_1 = -11$
$x_2 = 3$
$x_3 = 0$

$$\vec{x} = \begin{bmatrix} -11 \\ 3 \\ 0 \end{bmatrix}$$

5. According to a theorem we looked at, does the equation $A\vec{x} = \vec{b}$ always have a solution for all $\vec{b} \in \mathbb{R}^4$ when $A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$?

equivalent to every row having a pivot

No!

$$\xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 3/2 & 0 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

a row with no pivot.

6. Do $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$ span $\mathbb{R}^4$?

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

a row with no pivot.

cols of my A <u>don't</u> span $\mathbb{R}^4$.