Date ____________

MTH 261
Linear Algebra
Spring 2017

Linear Transformations

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

$T_A : \mathbb{R}^3 \to \mathbb{R}^3$

Maybe

1. Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -3 \\ 2 & -5 & 6 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} -6 \\ -4 \\ -5 \end{bmatrix}$. Is $\vec{b}$ in the image (range) of T_A?

If... $A\vec{x} = \vec{b}$ has a solution?

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -6 \\ 0 & 1 & -3 & -4 \\ 2 & -5 & 6 & -5 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & -6 \\ 0 & 1 & -3 & -4 \\ 0 & -1 & 0 & 7 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} \boxed{1} & -2 & 3 & -6 \\ 0 & \boxed{1} & -3 & -4 \\ 0 & 0 & \boxed{-3} & 3 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & -2 & 0 & -3 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & -1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -17 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

Yes, it does have a solution. So yes, $\vec{b}$ is in the image of T_A.

input vectors have 3 entries

output vectors have 4 entries

2. Let $A = \begin{bmatrix} 1 & -3 & 2 \\ 3 & -8 & 8 \\ 0 & 1 & 2 \\ 1 & 0 & 8 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$, and $\vec{c} = \begin{bmatrix} 1 \\ 6 \\ 3 \\ 10 \end{bmatrix}$. Is $\vec{b}$ in the image of T_A? Is $\vec{c}$ in the image of T_A? What is $T_A\left(\vec{b}\right)$?

No! $\vec{b}$ is in $\mathbb{R}^3$, but the codomain is $\mathbb{R}^4$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ 3 & -8 & 8 & 6 \\ 0 & 1 & 2 & 3 \\ 1 & 0 & 8 & 10 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 8 & 10 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

System is consistent. There is a solution to $A\vec{x} = \vec{c}$. So $\vec{c}$ is in the image of T_A.

$$T_A(\vec{b}) = A \cdot \vec{b} = 2\begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \end{bmatrix} + 1 \cdot \begin{bmatrix} -3 \\ -8 \\ 1 \\ 0 \end{bmatrix} - 3\begin{bmatrix} 2 \\ 8 \\ 2 \\ 8 \end{bmatrix} = \begin{bmatrix} -7 \\ -26 \\ -5 \\ -22 \end{bmatrix}$$

3. Define $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ by $T(\vec{x}) = \begin{bmatrix} x_2 - x_1 \\ 3x_1 + x_2 \\ x_1 \end{bmatrix}$. Is T a linear transformation? If so, prove it according to the definition of a linear transformation. If not, show why not.

$$T(\vec{u}+\vec{v}) \stackrel{?}{=} T(\vec{u}) + T(\vec{v})$$

$$T\left(\begin{bmatrix} u_1+v_1 \\ u_2+v_2 \end{bmatrix}\right)$$

$$\begin{bmatrix} u_2+v_2 - (u_1+v_1) \\ 3(u_1+v_1) + (u_2+v_2) \\ u_1+v_1 \end{bmatrix} \stackrel{?}{=} \checkmark \begin{bmatrix} u_2 - u_1 \\ 3u_1 + u_2 \\ u_1 \end{bmatrix} + \begin{bmatrix} v_2 - v_1 \\ 3v_1 + v_2 \\ v_1 \end{bmatrix}$$

$$T(c\vec{u}) \stackrel{?}{=} cT(\vec{u})$$

$$T\left(\begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix}\right) \stackrel{?}{=} c \cdot \begin{bmatrix} u_2 - u_1 \\ 3u_1 + u_2 \\ u_1 \end{bmatrix}$$

$$\begin{bmatrix} cu_2 - cu_1 \\ 3cu_1 + cu_2 \\ cu_1 \end{bmatrix} \stackrel{?}{=} \checkmark \begin{bmatrix} c(u_2-u_1 \\ c(3u_1+u_2 \\ cu_1 \end{bmatrix}$$

4. Define $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ by $T(\vec{x}) = \begin{bmatrix} x_3 + 1 \\ x_1 - x_2 \end{bmatrix}$. Is T a linear transformation? If so, prove it according to the definition of a linear transformation. If not, show why not.

$$T(\vec{0}) = T\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$T(\vec{0}) =$ not $\vec{0}$

$\Longrightarrow T$ is not linear.

5. Define $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ by $T(\vec{x}) = \begin{bmatrix} |x_1| \\ x_1 + x_2 \\ \sin(x_3) \end{bmatrix}$. Is T a linear transformation? If so, prove it according to the definition of a linear transformation. If not, show why not.

Very suspicious.. suspect it's <u>not</u> linear.

Note $$T\left(-1 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) \stackrel{?}{=} -1 \cdot T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$$

$$T\left(\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}\right) \stackrel{?}{=} -1 \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

No!