Date ____________

MTH 261
Linear Algebra
Spring 2017

Linear Independence in $\mathbb{R}^n$

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Determine if the set of vectors is linearly independent. You may be able to do this quikly by citing a theorem or a fact we established, or you may need more investigation.

(a) $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \end{bmatrix} \right\}$

Independent since the second vector is clearly not a multple of the first.

(b) $\left\{ \begin{bmatrix} -2 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ -6 \end{bmatrix} \right\}$

Dependent since $\vec{V}_2 = \frac{-3}{2}\vec{V}_1$

(c) $\left\{ \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 7 \end{bmatrix} \right\}$

$$\left[\begin{array}{ccc|c} -2 & 2 & -2 & 0 \\ 1 & -2 & 0 & 0 \\ 3 & 1 & 7 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

its linearly dependent

(d) $\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} \right\}$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & -2 & 2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

its linearly <u>in</u>dependent

2. In each problem above, if the set was linearly dependent, demonstrate the dependency explicitly.

$$\frac{3}{2}\vec{V}_1 + 1\,\vec{V}_2 = \vec{0}$$

c_3 free.
$c_2 = -c_3$
$c_1 = -2c_3$

can choose
$c_3 = 1$
$c_2 = -1$
$c_1 = -2$

$$-2\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} + (-1)\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} + 1\begin{bmatrix} -2 \\ 0 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

3. Suppose that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a linearly independent set. Use this to determine if $\{\vec{v}_1, \vec{v}_1 + \vec{v}_2, \vec{v}_1 + \vec{v}_2 + \vec{v}_3\}$ is also a linearly independent set.

is independent?

$$c_1\vec{V}_1 + c_2(\vec{v}_1+\vec{v}_2) + c_3(\vec{v}_1+\vec{v}_2+\vec{v}_3) = \vec{0}$$

(does this have nontrivial solutions?)

$$\underbrace{(c_1+c_2+c_3)}\vec{v}_1 + \underbrace{(c_2+c_3)}\vec{v}_2 + \underbrace{c_3}\vec{v}_3 = \vec{0}$$

there ~~are~~ no nontrivial k_1, k_2, k_3

Solving this. $k_1 = k_2 = k_3 = 0$

~~At least one $k \neq 0$ $\Longrightarrow$ this system has a nonzero sol for c_i's.~~

possible for

$c_1 + c_2 + c_3 = k_1$

$c_2 + c_3 = k_2$

$c_3 = k_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & k_1 \\ 0 & 1 & 1 & k_2 \\ 0 & 0 & 1 & k_3 \end{array}\right]$$

$\Downarrow$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\Downarrow$

$c_1 = 0$

$c_2 = 0$

$c_3 = 0$

$\Downarrow$

By def of linear independence

$\{\vec{v}_1, \vec{v}_1+\vec{v}_2, \vec{v}_1+\vec{v}_2+\vec{v}_3\}$

is independent

4. How many pivot columns must a 12 × 7 matrix have if its columns are linearly independent? Why?

$\Downarrow$

all scalars c_i would have to be 0.

$\Longrightarrow$ there has to be 7 pivots.

5. Make a set of three vectors in $\mathbb{R}^3$ with all nonzero entries that you know are linearly dependent. One way to do this is to make up the first two vectors and then linearly combine them in some way to get a third.

$$\left\{ \begin{array}{c} 2 \\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \end{array}, \begin{array}{c} -3 \\ \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \end{array}, \begin{bmatrix} -10 \\ -11 \\ -12 \end{bmatrix} \right\}$$

✓ is dependent

If these vectors were columns of a matrix A, would the equation $A\vec{x} = \vec{0}$ be consistent? yes
Would it have a unique solution?

no.

$$2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + (-3)\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + (-1)\begin{bmatrix} -10 \\ -11 \\ -12 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$