Date ___________

MTH 261
Linear Algebra
Spring 2017

The Invertible Matrix Theorem

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Use the Invertible Matrix Theorem to explain why: if A is invertible, then the columns of A^T are linearly independent. (This takes two steps of explanation.)

A invertible $\Rightarrow$ A^T invertible (by item l)
$\Rightarrow$ A^T's columns are linearly independent (by item e)

2. Can a square matrix with two parallel *rows* be invertible? Cite specifics in the Invertible Matrix Theorem that back up your answer.

No. Because then A^T would have two linearly dependent columns, and by item e, A^T would not be invertible. So by item l, A would not be invertible.

3. Suppose that two $n \times n$ matrices E and F are such that $EF = I$. Citing the Invertible Matrix Theorem when appropriate, can we conclude that $EF = FE$? Answer using logic and *only* the items from the Invertible Matrix Theorem.

If $EF = I$, then we know E is invertible by item j. By item k, there exists D with $DE = I$. Now right-multiply by F: $(DE)F = IF$
$\Rightarrow D(EF) = F$
$\Rightarrow DI = F$
$\Rightarrow D = F$

So since $DE = I$, we have $FE = I$.
So $EF = I = FE$.

4. Let A and B be $n \times n$ matrices. If AB is an invertible matrix, is B necessarily invertible? Cite the Invertible Matrix Theorem appropriately.

Yes.

$(AB)^{-1}(AB) = I$ (justified because AB is invertible.)

So $[(AB)^{-1}A]\,B = I$. So by item k, B is invertible.

5. Suppose that $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ are linearly independent vectors in $\mathbb{R}^n$. Let

$$A = \begin{bmatrix} \text{---} & \vec{v}_1^T & \text{---} \\ \text{---} & \vec{v}_2^T & \text{---} \\ & \vdots & \\ \text{---} & \vec{v}_n^T & \text{---} \end{bmatrix}$$

Is A necessarily invertible? Why or why not?

Well, A^T is $\begin{bmatrix} | & | & & | \\ \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_n \\ | & | & & | \end{bmatrix}$. Since A^T has linearly indep. columns, by item e, A^T is invertible. By item l, A must be invertible.

6. Without looking, try to write down as many of the parts of the Invertible Matrix Theorem as you can recall. At this point, there are 12 parts.