Date _____

MTH 261
Linear Algebra
Spring 2017

Solution Sets to Homogeneous and Nonhomogeneous Linear Systems

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Suppose that the solution set of a system of linear equations can be described as

   $x_1 = 4 + 5x_3$
   $x_2 = -4 - 6x_3$
   x_3 is free

   $$\vec{x} = \begin{bmatrix} 4+5x_3 \\ -4-6x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -6 \\ 1 \end{bmatrix}$$

   The solution set is a line in $\mathbb{R}^3$ passing through the point $\langle 4, -4, 0 \rangle$ and parallel to the vector $\langle 5, -6, 1 \rangle$.

2. Find all solutions to the system and write them in *parametric* form.

   $2x_1 - 3x_2 - 4x_3 = 4$
   $-2x_1 - 3x_2 + 4x_3 = -4$
   $5x_2 - 5x_3 = 7$

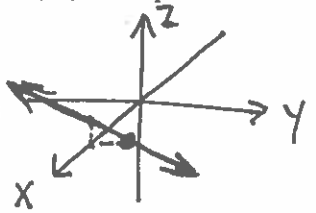
   $$\begin{bmatrix} 2 & -3 & -4 & 4 \\ -2 & 3 & 4 & -4 \\ 0 & 5 & 5 & 7 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & -3 & -4 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 5 & 5 & 7 \end{bmatrix}$$

   $$\longrightarrow \begin{bmatrix} 2 & -3 & -4 & 4 \\ 0 & 1 & 1 & 7/5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 0 & -1 & 41/5 \\ 0 & 1 & 1 & 7/5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   $$\longrightarrow \begin{bmatrix} 1 & 0 & -1/2 & 41/10 \\ 0 & 1 & 1 & 7/5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 41/10 + \frac{1}{2}x_3 \\ 7/5 - x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 41/10 \\ 7/5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1/2 \\ -1 \\ 1 \end{bmatrix}$$

   $$\text{Sol set} = \left\{ \begin{bmatrix} 41/10 \\ 7/5 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1/2 \\ -1 \\ 1 \end{bmatrix} \;\middle|\; t \in \mathbb{R} \right\}$$

   Once you have done the above:
   The solution set is a line in $\mathbb{R}^3$ passing through the point $\langle 41/10, 7/5, 0 \rangle$ and parallel to the vector $\langle 1/2, -1, 1 \rangle$. Try to sketch the solution set.



3. *Using work you already did in problem 2*, find all solutions to the system below in parametric form.

$$\begin{aligned} 2x_1 - 3x_2 - 4x_3 &= 0 \\ -2x_1 - 3x_2 + 4x_3 &= 0 \\ 5x_2 + 5x_3 &= 0 \end{aligned}$$

$$\left\{ t\begin{bmatrix} 1/2 \\ -1 \\ 1 \end{bmatrix} \;\middle|\; t \in \mathbb{R} \right\}$$

4. Find a parametric equation of the line through $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$ parallel to $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

$$\begin{bmatrix} 1 \\ 5 \end{bmatrix} + t\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

5. Describe all solutions to $A\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix}$ in parametric form, where

$$A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \left[\begin{array}{cccccc|c} \boxed{1} & -4 & -2 & 0 & 3 & -5 & 1 \\ 0 & 0 & \boxed{1} & 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 & \boxed{1} & -4 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

RREF $\left[\begin{array}{cccccc|c} \boxed{1} & -4 & 0 & 0 & 0 & 5 & -4 \\ 0 & 0 & \boxed{1} & 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 & \boxed{1} & -4 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$ → if free variables are all set to 0, $x_1 = -4$

if free vars are set to 0,

$x_3 = 2$

$x_5 = 3$

$$\vec{x} = \begin{bmatrix} * \\ * \\ * \\ * \\ * \\ * \end{bmatrix} = \begin{bmatrix} -4 + \cdots \\ * \\ 2 + \cdots \\ * \\ 3 + \cdots \\ * \end{bmatrix}$$

$x_1 = -4 + 4x_2 - 5x_6$

x_2 is free

$x_3 = 2 + x_6$

x_4 is free

$x_5 = 3 + 4x_6$

x_6 free

$$\vec{x} = \begin{bmatrix} (-4) + 4x_2 - 5x_6 \\ x_2 \\ (2) + x_6 \\ x_4 \\ (3) + 4x_6 \\ x_6 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 2 \\ 0 \\ 3 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6\begin{bmatrix} -5 \\ 0 \\ 1 \\ 0 \\ 4 \\ 1 \end{bmatrix}$$