Date _____

MTH 261
Linear Algebra
Spring 2017
Echelon Form

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Determine if these matrices are in (unreduced) row echelon form [REF], reduced row echelon form [RREF], or none of the above.

$\begin{bmatrix}1&0&2\\0&1&3\end{bmatrix}$ RREF

$\begin{bmatrix}1&2&0\\0&0&1\end{bmatrix}$ RREF

$\begin{bmatrix}1&2&3\\0&1&2\\0&0&1\end{bmatrix}$ REF (not RREF)

$\begin{bmatrix}1&2&0\\0&0&2\\0&0&0\end{bmatrix}$ REF (not RREF)

$\begin{bmatrix}1&2&0&-1\\0&0&1&-2\\0&0&0&0\end{bmatrix}$ RREF

$\begin{bmatrix}1&0&0&4&-1\\0&0&1&5&2\\0&1&0&0&-1\end{bmatrix}$ none of the above because leading entries are positioned like ↘ instead of downward and rightward

$\begin{bmatrix}1&0&-3&4\\0&1&1&5\\0&0&0&0\end{bmatrix}$ RREF

$\begin{bmatrix}1&0&4&2/3\\0&1&5&6\\0&0&1&1/3\end{bmatrix}$ REF (not RREF)

2. Row reduce these matrices into reduced row echelon form. Circle the pivot positions in the original matrix. Which columns are pivot columns?

$\begin{bmatrix}1&2&3&4\\5&6&7&8\\9&10&11&12\end{bmatrix}$

$-5R_1+R_2\to R_2$, $-9R_1+R_3\to R_3$ $\begin{bmatrix}1&2&3&4\\0&-4&-8&-12\\0&-8&-16&-24\end{bmatrix}$

pivot columns

$-2R_2+R_3\to R_3$, $-\frac{1}{4}R_2\to R_2$ $\begin{bmatrix}1&2&3&4\\0&1&2&3\\0&0&0&0\end{bmatrix}$

$-2R_2+R_1\to R_1$ $\begin{bmatrix}1&0&-1&-2\\0&1&2&3\\0&0&0&0\end{bmatrix}$

$\begin{bmatrix}1&2&3&5\\2&3&5&7\\3&5&7&11\end{bmatrix}$

pivot columns

$-2R_1+R_2\to R_2$, $-3R_1+R_3\to R_3$ $\begin{bmatrix}1&2&3&5\\0&-1&-1&-3\\0&-1&-2&-4\end{bmatrix}$

$-R_2+R_3\to R_3$, $-R_2\to R_2$ $\begin{bmatrix}1&2&3&4\\0&1&1&3\\0&0&-1&-1\end{bmatrix}$

$R_3+R_2\to R_2$, $3R_3+R_1\to R_1$ $\begin{bmatrix}1&2&0&1\\0&1&0&2\\0&0&-1&-1\end{bmatrix}$

$-2R_2+R_1\to R_1$, $-R_3\to R_3$ $\begin{bmatrix}1&0&0&-3\\0&1&0&2\\0&0&+1&+1\end{bmatrix}$

3. Suppose that a 3×5 *coefficient* matrix for a system has three pivot columns. Is the system necessarily consistent or inconsistent? Why?

The matrix only has 3 rows. So every row has a pivot position. Since this is the coefficient matrix, the RREF version of the matrix corresponds to equations where every variable is either free, or can be expressed in terms of free variables.

4. Suppose that a 3×5 *augmented* matrix for a system has its fifth column as a pivot column. Is the system necessarily consistent or inconsistent? Why?

It's inconsistent. If a pivot position is in the last column, then that row corresponds to the equation

$$0x_1 + 0x_2 + 0x_3 + 0x_4 = 1$$

which has no solutions.

5. Suppose the coefficient matrix of a system has a pivot in every row. Explain why the system is consistent.

Basically, same answer as for #3.

6. Suppose the augmented matrix of a system has a pivot in every column except the last one. Explain why the system is not only consistent, but has a unique solution.

Since there is a pivot for every variable, each variable can be solved for directly in the RREF version of the matrix. Furthermore, there are no free variables, since there are no columns which are not pivot columns. So there is a unique solution.