Date _____

MTH 261
Linear Algebra
Spring 2017
Determinant Computation

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Find the determinant of $\begin{bmatrix} 8 & -9 \\ 2 & -4 \end{bmatrix}$ using the special 2×2 determinant formula.

$8(-4) - (-9)(2) = -32 + 18 = -14$

2. Use the special 3×3 determinant rule to find $\begin{vmatrix} 2 & 1 & -5 \\ 2 & 2 & -4 \\ 1 & 3 & 1 \end{vmatrix}$.

$= 2\cdot 2\cdot 1 + 1\cdot(-4)\cdot 1 + (-5)\cdot 2\cdot 3 - 2(-4)\cdot 3 - 1\cdot 2\cdot 1 - (-5)\cdot 2\cdot 1$

$= 4 - 4 - 30 + 24 - 2 + 10$

$= 0 - 6 + 8$

$= 2$

3. Use the (hyper)-volume definition of the determinant function to find the determinant of $\begin{bmatrix} 2 & 4 & 6 & 8 \\ 1 & 2 & -3 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & 2 \end{bmatrix}$

← parallel rows ⟹ hyperthy is "degenerate" it's not really 4D.

has hypervolume 0.

So the determinate is 0.

4. Use row reduction to find $\begin{vmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & 4 & 8 \\ 0 & 2 & 4 & -3 \\ 1 & -2 & -2 & -9 \end{vmatrix}$

$A \xrightarrow[\text{no effect}]{} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & 4 & 8 \\ 0 & 2 & 4 & -3 \\ 0 & 0 & -3 & -9 \end{bmatrix} \xrightarrow{\times \frac{1}{2}} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 4 \\ 0 & 2 & 4 & -3 \\ 0 & 0 & -3 & -9 \end{bmatrix}$

$\xrightarrow[\text{no effect}]{} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & -11 \\ 0 & 0 & -3 & -9 \end{bmatrix} \xrightarrow[\times -\frac{1}{3}]{\times -\frac{1}{11}} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix} \xrightarrow{\text{recycle}} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \to I$

$-\frac{66}{11} \leftarrow \times 2 \leftarrow \times(-\tfrac{1}{11}) \times(-3) \leftarrow \times(-1) \leftarrow \det = 1$

5. Find the determinant of $\begin{bmatrix} 2 & 1 & 1 & 1 \\ 2 & -1 & 0 & 1 \\ 1 & 0 & 0 & -4 \\ 0 & 1 & -2 & 3 \end{bmatrix}$ by expanding across a row or column.

$1 \cdot (-1)^{3+1} \begin{vmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 3 \end{vmatrix} + 4 (-1)^{3+4} \begin{vmatrix} 2 & 1 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & -2 \end{vmatrix}$

$\underbrace{1\left(0+1+2-(-2)-(-3)-0\right)}_{8} + 4(-1)\left(4+0+2-0-(-4)-0\right)$

$-4(10) = -32$

6. Find the determinant of $\begin{bmatrix} 2 & -3 & 0 & 0 \\ 1 & 0 & 4 & 5 \\ 0 & -2 & -2 & 0 \\ -1 & 0 & 0 & 4 \end{bmatrix}$ using the permutations-based definition of the determinant.

permutations that contribute nonzero:

$\begin{bmatrix} \cdot & & \cdot \\ & \cdot & \\ & & \cdot \end{bmatrix} \begin{bmatrix} & \cdot & \\ \cdot & & \cdot \\ & & \end{bmatrix} \begin{bmatrix} & & -3 & \\ & & & 5 \\ & -2 & & \\ -1 & & & \end{bmatrix}$

odd odd even

$-2 \cdot 4 (-2) \cdot 4 - (-3)(1)(-2)(4) + (-3)(5)(-2)(-1)$

$64 - 24 - 30$

$= 14$