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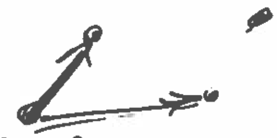
MTH 261
Linear Algebra
Spring 2017
Determinant Applications



(0,0,0)

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Find the area of the parallelogram in $\mathbb{R}^3$ with vertices at $(0,0,0)$, $(1,2,-1)$, $(2,-3,-2)$, and $(3,-1,-3)$.

$\langle 1,2,-1\rangle$

Fact: $\|\vec{u} \times \vec{v}\|$ gives area of a parallelogram where $\vec{u}, \vec{v}$ are adjacent side vectors.

$\langle 2,-3,-2\rangle$

$$\langle 1,2,1\rangle \times \langle 2,-3,-2\rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -3 & -2 \end{vmatrix} = -4\hat{i} + 2\hat{j} - 3\hat{k} - (+3)\hat{i} - (-2\hat{j}) - 4\hat{k}$$

$$= -7\hat{i} - 7\hat{k} = \langle -7,0,-7\rangle$$

has magnitude $7\sqrt{2}$.

2. Find the area of the parallelogram in $\mathbb{R}^2$ with vertices at $(0,0)$, $(3,2)$, $(4,-7)$, and $(7,-5)$. ~~(Hint: embed $\mathbb{R}^2$ in $\mathbb{R}^3$ by identifying the xy plane in $\mathbb{R}^3$ with $\mathbb{R}^2$ in the simplest way.~~

$\langle 3,2\rangle$ $\langle 4,-7\rangle$

$$\det\begin{bmatrix} 3 & 2 \\ 4 & -7 \end{bmatrix} = -21 - 8 = -29$$

$$|-29| = 29.$$

3. A linear transformation T has standard matrix $A_T = \begin{bmatrix} 1 & 2 & -4 \\ 0 & 4 & 2 \\ 3 & -5 & 10 \end{bmatrix}$. If C is the standard unit cube in $\mathbb{R}^3$, what will the volume of $T(C)$ be?

$$40 + 12 + 0 - (-10) - 0 - (-48)$$

$$= 52 \quad + 10 \quad + 48 \quad = 110$$

4. Find $\vec{u} \times \vec{v}$, where $\vec{u} = \langle 5, 10, 12 \rangle$ and $\vec{v} = \langle 4, -3, 8 \rangle$.

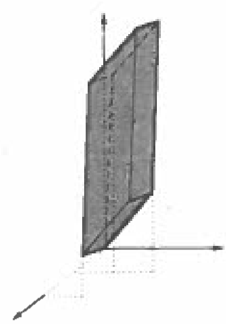
$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & 10 & 12 \\ 4 & -3 & 8 \end{vmatrix} = 80\hat{\imath} + 48\hat{\jmath} - 15\hat{k} - (-36\hat{\imath}) - 40\hat{\jmath} - 40\hat{k}$$

$$= 116\hat{\imath} + 8\hat{\jmath} - 55\hat{k}$$

$$= \langle 116, 8, -55 \rangle$$

5. Consider the parallelepiped P (in the figure) with vertices at

$$\begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}2\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}1\\1\\5\end{bmatrix}, \begin{bmatrix}3\\3\\3\end{bmatrix}, \begin{bmatrix}2\\3\\7\end{bmatrix}, \begin{bmatrix}3\\2\\6\end{bmatrix}, \begin{bmatrix}4\\4\\8\end{bmatrix}.$$



Find the volume enclosed by P.

$$\det \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 5 \end{bmatrix} = 20 + 2 + 1 - 4 - 5 - 2$$

$$= 22 - 3 - 7$$

$$= 12$$