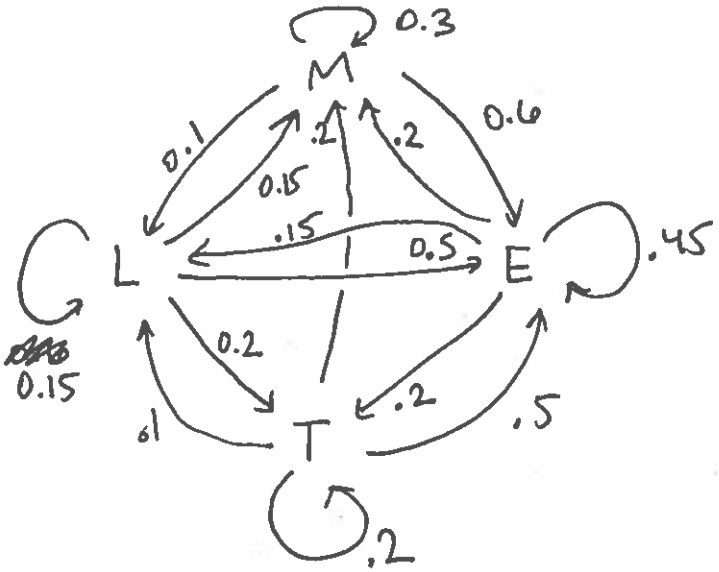
Date _____

MTH 261
Linear Algebra
Spring 2017
Applications of Linear Systems

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Suppose an economy has four sectors: Mining, Lumber, Energy, and Transportation. Mining sells 10% of its output to lumber, 60% to Energy, and retains the rest. Lumber sells 15% of its output to Mining, 50% to Energy, 20% to Transportation, and retains the rest. Energy sells 20% of its output to Mining, 15% to Lumber, 20% to Transportation, and retains the rest. Transportation sells 20% of its output to Mining, 10% to Lumber, 50% to Energy, and retains the rest.

   If the total value of this economy is 1 billion dollars, what outputs from each sector make the economy in balance?



$$P_M = 0.3P_M + 0.15P_L + 0.2P_E + 0.2P_T$$
$$0 = -0.7P_M + 0.15P_L + 0.2P_E + 0.2P_T$$

$$P_L = 0.1P_M + 0.15P_L + 0.15P_E + 0.1P_T$$
$$0 = 0.1P_M - 0.85P_L + 0.15P_E + 0.1P_T$$

$$P_E = 0.6P_M + 0.5P_L + 0.45P_E + 0.5P_T$$
$$0 = 0.6P_M + 0.5P_L - 0.55P_E + 0.5P_T$$

$$P_T = 0.2P_L + 0.2P_E + 0.2P_T$$
$$0 = 0.2P_L + 0.2P_E - 0.8P_T$$

$$P_M + P_L + P_E + P_T = 1$$

$$\left[\begin{array}{cccc|c} -.7 & .15 & .2 & .2 & 0 \\ .1 & -.85 & .15 & .1 & 0 \\ .6 & .5 & -.55 & .5 & 0 \\ 0 & .2 & .2 & -.8 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

use tech to RREF

$P_M = \frac{193}{898} \approx 0.215$ ($B billion)
$P_E = \frac{59}{449} \approx 0.131$ ($B)
$P_E = \frac{223}{449} \approx 0.497$ ($B)
$P_T = \frac{141}{898} \approx 0.157$ ($B)

2. Use linear algebra to balance this chemical reaction equation.

$$\underset{n_1}{PbN_6} + \underset{n_2}{CrMn_2O_8} \rightarrow \underset{n_3}{Pb_3O_4} + \underset{n_4}{Cr_2O_3} + \underset{n_5}{MnO_2} + \underset{n_6}{NO}$$

Pb: $n_1 = 3n_3$
N: $6n_1 = n_6$
Cr: $n_2 = 2n_4$
Mn: $2n_2 = n_5$
O: $8n_2 = 4n_3 + 3n_4 + 2n_5 + n_6$

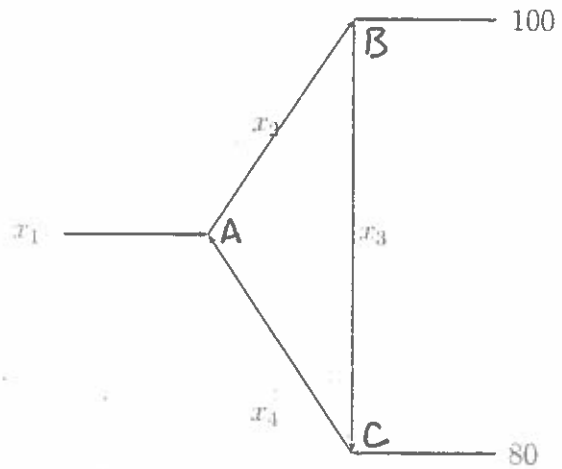
$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & 0 & 0 & -22/45 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & 0 & -11/45 & 0 \\ 0 & 0 & 0 & 0 & 1 & -44/45 & 0 \end{bmatrix}$$

$n_1 = 1/6\, n_6$ $\quad n_4 = 11/45\, n_6$
$n_2 = 22/45\, n_6$ $\quad n_5 = 44/45\, n_6$
$n_3 = 1/18\, n_6$

Let $n_6 = 90$. $\quad n_1 = 15$, $n_2 = 44$, $n_3 = 5$, $n_4 = 22$, $n_5 = 88$

So $15PbN_6 + 44CrMn_2O_8 \rightarrow 5Pb_3O_4 + 22Cr_2O_3 + 88MnO_2 + 90NO$

3. Find the general flow pattern of the traffic network below. Assume that all flows are non-negative. What is the range of values x_3 can take?



A: $x_1 + x_4 = x_2$
B: $x_2 = 100 + x_3$
C: $x_3 + 80 = x_4$

$$\begin{cases} x_1 - x_2 + x_4 = 0 \\ x_2 - x_3 = 100 \\ x_3 - x_4 = -80 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 100 \\ 0 & 0 & 1 & -1 & -80 \end{bmatrix}$$

$R_3 + R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 20 \\ 0 & 0 & 1 & -1 & -80 \end{bmatrix}$$

$R_2 + R_1 \rightarrow R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & -1 & 20 \\ 0 & 0 & 1 & -1 & -80 \end{bmatrix}$$

x_4 is free $\longrightarrow x_4 \geq 0$
$x_3 = x_4 - 80 \longrightarrow x_4 \leq 80$
$x_2 = x_4 + 20 \longrightarrow x_4 \geq -20$ (redundant)
$x_1 = 20$

x_4 ranges 0 to 80
x_3 ranges 0 to 80
x_2 ranges 20 to 100
$x_1 = 20$.