Date ____________

MTH 261
Linear Algebra
Summer 2017

The Matrix of a Linear Transformation

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Let T from $\mathbb{R}^2$ to $\mathbb{R}^3$ be such that

$$T : \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x+y \\ 2x+y \\ 3x-y \end{bmatrix}$$

   (a) Find the matrix for T with respect to the standard coordinate vectors, A_T.

   (b) Find $T\left(\begin{bmatrix} 2 \\ -2 \end{bmatrix}\right)$ in two ways: by directly using the rule for T and by using the matrix A_T.

2. Identify the space of 2×2 matrices with $\mathbb{R}^4$ by identifying each matrix in

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

   with a coordinate vector from $\mathbb{R}^4$. Let T be the linear transformation from $M_{2\times 2}$ to $\mathbb{R}^2$ given by $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \mapsto \begin{bmatrix} a+d \\ b-c \end{bmatrix}$.

   (a) Find A_T. (A_T is a matrix — do you understand what dimensions it should have?)

   (b) Find $T\left(\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\right)$ in two ways: by directly using the rule for T and by using the matrix A_T.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the operator that rotates $\mathbb{R}^2$ about the origin counterclockwise by an angle of $45°$.

   (a) Find the matrix for T with respect to the standard basis.

   (b) Find $T\left(\begin{bmatrix} 10 \\ 2 \end{bmatrix}\right)$.

4. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first rotates vectors about the origin by $90°$ clockwise, and then reflects perpendicularly through the line ℓ.

   (a) Find A_T, the standard matrix for T.

   (b) Find $T\left(\begin{bmatrix} 13 \\ 4 \end{bmatrix}\right)$.