

MTH 261

LINEAR ALGEBRA

SUMMER 2017

The Matrix Equation $A\vec{x} = \vec{b}$

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Compute the products all the way. Start by rewriting the product as a linear combination of vectors. Watch out — I may be trying to trick you.

$$\begin{bmatrix} -4 & 2 \\ 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ -3 & 1 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

2. Without simplifying anything, rewrite the equation as a vector equation.

$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ -2 & -3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$

3. Rewrite this vector equation as a matrix equation.

$$x_1 \begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$

4. Solve the equation $\begin{bmatrix} 1 & 3 & -4 \\ 1 & 5 & 2 \\ -3 & -7 & 6 \end{bmatrix} \vec{x} = \begin{bmatrix} -2 \\ 4 \\ 12 \end{bmatrix}$. If this equation doesn't quite make sense to you, try rewriting it as a vector equation or as a system of linear equations.

5. According to a theorem we looked at, does the equation $A\vec{x} = \vec{b}$ always have a solution for all $\vec{b} \in \mathbb{R}^4$ when $A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$?

6. Do $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$ span $\mathbb{R}^4$? Don't take a "shortcut" based on what you think you know about dimension. To literally answer what is asked, use a theorem that we looked at.