Date ____________

MTH 261
Linear Algebra
Summer 2017

Linear Independence in $\mathbb{R}^n$

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Determine if the set of vectors is linearly independent. You may be able to do this quikly by citing a theorem or a fact we established, or you may need more investigation.

   (a) $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \end{bmatrix} \right\}$

   (b) $\left\{ \begin{bmatrix} -2 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ -6 \end{bmatrix} \right\}$

   (c) $\left\{ \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 7 \end{bmatrix} \right\}$

   (d) $\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} \right\}$

2. In each problem above, if the set was linearly dependent, demonstrate the dependency explicitly.

3. Suppose that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a linearly independent set. Use this to determine if $\{\vec{v}_1, \vec{v}_1 + \vec{v}_2, \vec{v}_1 + \vec{v}_2 + \vec{v}_3\}$ is also a linearly independent set.

4. How many pivot columns must a 12×7 matrix have if its columns are linearly independent? Why?

5. Make a set of three vectors in $\mathbb{R}^3$ with all nonzero entries that you know are linearly dependent. One way to do this is to make up the first two vectors and then linearly combine them in some way to get a third.

   If these vectors were columns of a matrix A, would the equation $A\vec{x} = \vec{0}$ be consistent? Would it have a unique solution?