MTH 261
Linear Algebra
Spring 2017
Echelon Form

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Determine if these matrices are in (unreduced) row echelon form [REF], reduced row echelon form [RREF], or none of the above.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 4 & -1 \\ 0 & 0 & 1 & 5 & 2 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 & 4 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 4 & {}^2\!/_3 \\ 0 & 1 & 1 & 5 & 6 \\ 0 & 0 & 0 & 1 & {}^1\!/_3 \end{bmatrix}$$

2. Row reduce these matrices into reduced row echelon form. Circle the pivot positions in the original matrix. Which columns are pivot columns?

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 3 & 5 & 7 \\ 3 & 5 & 7 & 11 \end{bmatrix}$$

3. Suppose that a 3×5 *coefficient* matrix for a system has three pivot columns. Is the system necessarily consistent or inconsistent? Why?

4. Suppose that a 3×5 *augmented* matrix for a system has its fifth column as a pivot column. Is the system necessarily consistent or inconsistent? Why?

5. Suppose the coefficient matrix of a system has a pivot in every row. Explain why the system is consistent.

6. Suppose the augmented matrix of a system has a pivot in every column except the last one. Explain why the system is not only consistent, but has a unique solution.