Date _____

MTH 261
Linear Algebra
Summer 2017
Column Space and Null Space

Find partners, and follow the instructions. You will not turn this in, but you must be working diligently to get attendance credit.

1. Find the null space and column space for $A = \begin{bmatrix} 1 & -2 & 2 & 2 \\ 0 & 3 & 1 & -1 \end{bmatrix}$. Specifically, I mean write Nul A and Col A as the span of some vector(s).

2. In the last problem you probably wrote Col A as the span of four vectors. Show that both $\vec{e}_1$ and $\vec{e}_2$ are in this span. (Note that $\vec{e}_i$ is one thing in the context of Col A and something different in the context of Nul A.) Does this mean Col $A = \text{Span}\{\vec{e}_1, \vec{e}_2\}$?

3. Find the null space and column space for $A = \begin{bmatrix} 1 & -2 & 3 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}$. Specifically, I mean write Nul A and Col A as the span of some vector(s).

4. Row reduce $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & -4 \end{bmatrix}$ into a reduced echelon matrix B. Do A and B have they have the same null space? Do they have the same column space?

5. Let $S = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a + b + c = 2 \right\}$. Is S a subspace of $\mathbb{R}^3$? Why or why not?

6. Let $A = \begin{bmatrix} 5 & 1 & 2 & 2 & 0 \\ 3 & 3 & 2 & -1 & -12 \\ 8 & 4 & 4 & -5 & 12 \\ 2 & 1 & 1 & 0 & -2 \end{bmatrix}$. Find a set of vectors that spans Nul A. Show that $\vec{a}_3$ and $\vec{a}_5$ are in Span$\{\vec{a}_1, \vec{a}_2, \vec{a}_4\}$. Explain why T_A is neither one-to-one nor onto. ('Onto' means that it's image is the entire codomain.)