5.2 continued:

Characteristic Polynomial of A.

Ex $A = \begin{bmatrix} 2 & 3 \\ 7 & 1 \end{bmatrix}$.

$\text{char}(A) = \det(A - \underset{\text{variable}}{\lambda} I)$

$\det\left(\begin{bmatrix} 2-\lambda & 3 \\ 7 & 1-\lambda \end{bmatrix}\right)$

$= (2-\lambda)(1-\lambda) - 21$

$= \lambda^2 - 3\lambda - 19$

roots:

$$\lambda = \frac{3 \pm \sqrt{9 - 4(1)(-19)}}{2} = \frac{3 \pm \sqrt{85}}{2}$$

are the eigenvalues...

There are nonzero vectors $\bar{x}_1$ and $\bar{x}_2$

where $A \cdot \bar{x}_1 = \left(\frac{3+\sqrt{85}}{2}\right) \cdot \bar{x}_1$

and $A \cdot \bar{x}_2 = \left(\frac{3-\sqrt{85}}{2}\right) \cdot \bar{x}_2$

We got

$X^2 - 3X - 19$

plug in $X = A$...

$A^2 - 3A - 19 \cdot I$

$\begin{bmatrix} 2 & 3 \\ 7 & 1 \end{bmatrix}^2 - 3\begin{bmatrix} 2 & 3 \\ 7 & 1 \end{bmatrix} - \begin{bmatrix} 19 & 0 \\ 0 & 19 \end{bmatrix}$

$\begin{bmatrix} 25 & 9 \\ 21 & 22 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 21 & 3 \end{bmatrix} - \begin{bmatrix} 19 & 0 \\ 0 & 19 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Fact: A matrix A, when plugged into its Char(A), results in the 0 matrix.

Additions to Invertible Matrix Theorem

(more things that are equivalent to "A is invertible")

(s) 0 is <u>not</u> an eigenvalue of A

(t) $\det(A) \neq 0$

(the only solution to $A\vec{x} = \vec{0}$ is $\vec{0}$)

<u>Def</u> "A is similar to B" means $A = P \cdot B \cdot P^{-1}$ for some invertible matrix P.

<u>Thm</u> If A, B are similar, A, B have the same characteristic polynomial, and the same eigenvalues...

<u>Warning!</u> Just because A, B have same eigenvalues A, B might not be similar

$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $\quad$ $B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

doubles all vectors in $\mathbb{R}^2$

$\lambda = 2$ is the only eigenvalue

$\lambda = 2$ is the only eigenvalue

<u>Not</u> similar despite having same eigenvalues.

... If $B = P \cdot A \cdot P^{-1}$

$B = P \cdot 2 \cdot I \cdot P^{-1} = 2 \cdot \underline{P \cdot I} \cdot P^{-1}$

$= 2 \cdot P \cdot P^{-1}$

Then $B = 2I$

this was a false assumption since $B \neq 2I$.

So B, A are not similar.

Section 5.3 Diagonalization.

Ex $A = \begin{bmatrix} 7 & -15 \\ 2 & -4 \end{bmatrix}$. Find A^{30}.

Even $A^2 = \begin{bmatrix} 7 & -15 \\ 2 & -4 \end{bmatrix} \cdot \begin{bmatrix} 7 & -15 \\ 2 & -4 \end{bmatrix}$

$= \begin{bmatrix} 7 \cdot 7 + -15 \cdot 2 & 7 \cdot (-15) - 15(-4) \\ 2 \cdot 7 - 4 \cdot 2 & 2 \cdot (-15) - 4(-4) \end{bmatrix}$

8 multiplications
4 additions

By the time A^{30}, that's a lot of arithmetic.

Well,

are inverses

$A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$

Then $A^k = (P \cdot D \cdot P^{-1})^k$

$= (PD\underbrace{P^{-1})(P}DP^{-1}) \cdots (PDP^{-1})$

$= P \cdot D \cdot D \cdots \cdot D \cdot P^{-1}$

$= P \cdot D^k \cdot P^{-1}$

So $A^k = P \cdot D^k \cdot P^{-1}$

$= \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}^k \begin{bmatrix} 2 & -3 \\ -1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2^k & 0 \\ 0 & 1^k \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -1 & 5 \end{bmatrix}$

$\lambda = 2$

$A\vec{x} = 2\vec{x}$

$(A - 2I)\vec{x} = \vec{0}$

$$\begin{bmatrix} 5 & -15 \\ 2 & -6 \end{bmatrix} \vec{x} = \vec{0}$$

take $x_2 = 1$
$x_1 = 3$

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

RREF

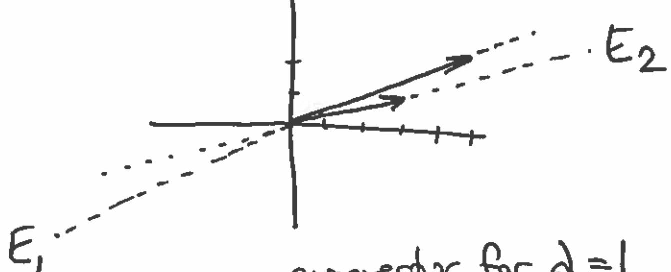
$$\left[\begin{array}{cc|c} 1 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

x_2 free

$x_1 = 3x_2$

We now have $\lambda = 2$ and $\lambda = 1$ ...

e'vector $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ e'vector $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$



eigenvector for $\lambda = 2$

eigenvector for $\lambda = 1$

So $\beta = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right\}$ forms a basis for $\mathbb{R}^2$.

came from different λ; theorem tells us e'vectors from distinct e'values are linearly independent.

Say you have $[\vec{x}]_\beta = \begin{bmatrix} p \\ q \end{bmatrix}$

Then $\left[T(\vec{x})\right]_\beta = \begin{bmatrix} 2p \\ q \end{bmatrix}$ so in β-coordinates, T's matrix $= \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

<u>Ex</u> Diagonalize $\begin{bmatrix} 7 & -15 \\ 2 & -4 \end{bmatrix}$

(Means Find $\underbrace{P}_{\text{invertible}}$, $\underbrace{D}_{\text{diagonal}}$ such that $A = P \cdot D \cdot P^{-1}$)

Find A's eigenvalues.

$\det(A - \lambda I) = 0$

$$\det\left(\begin{bmatrix} 7-\lambda & -15 \\ 2 & -4-\lambda \end{bmatrix}\right) = 0$$

$$(7-\lambda)(-4-\lambda) + 30 = 0$$

$$\lambda^2 - 3\lambda - 28 + 30 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(\lambda - 1)(\lambda - 2) = 0$$

$\lambda = 1, \quad \lambda = 2$

Gives you D... if you have as many e'values as n was in the n×n matrix.

$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ or $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

choose this to match earlier example...

<u>For each</u> eigenvalue, there is an eigenvector... find <u>one</u> eigenvector...

$\lambda = 1$

$A\bar{x} = 1 \cdot \bar{x}$

$(A - 1 \cdot I)\bar{x} = \vec{0}$

$\begin{bmatrix} 6 & -15 \\ 2 & -5 \end{bmatrix} \vec{x} = \vec{0}$

RREF $\left[\begin{array}{cc|c} 1 & -2.5 & 0 \\ 0 & 0 & 0 \end{array}\right]$

x_2 free

$x_1 = 2.5 x_2$

take $x_2 = 1$, $x_1 = 2.5$: $\begin{bmatrix} 2.5 \\ 1 \end{bmatrix}$

or

take $x_2 = 2$, $x_1 = 5$: $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

Consider letting $P = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$

e'vector for $\lambda=2$

e'vector for $\lambda=1$

This is $\underset{std \leftarrow \beta}{P}$

Now consider $\underbrace{\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}}_{\underset{std \leftarrow \beta}{P}} \underbrace{\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}^{-1}}_{\underset{\beta \leftarrow std}{P}}$

Then $\underbrace{\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \underbrace{\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}^{-1} \cdot [\vec{x}]_{std}}_{[\vec{x}]_\beta}}_{[A \cdot \vec{x}]_\beta}}_{[A \cdot \vec{x}]_{std}}$

$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}[\vec{x}]_\beta$ does the action that A was supposed to do...
stretching in $\vec{b}_1$ direction by 2
in $\vec{b}_2$ direction by 1

So $\boxed{A = P \cdot D \cdot P^{-1}}$

Final answer:

$$\begin{bmatrix} 7 & -15 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

Ex Diagonalize $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$

Find λ: $\det\left(\begin{bmatrix} 1-\lambda & 3 \\ 4 & 5-\lambda \end{bmatrix}\right) = 0$

$$(1-\lambda)(5-\lambda) - 12 = 0$$
$$\lambda^2 - 6\lambda + 5 - 12 = 0$$
$$\lambda^2 - 6\lambda - 7 = 0$$
$$(\lambda - 7)(\lambda + 1) = 0 \implies \lambda = 7, -1$$

Take $D = \begin{bmatrix} 7 & 0 \\ 0 & -1 \end{bmatrix}$

$\lambda = 7$

$$\begin{bmatrix} -6 & 3 \\ 4 & -2 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & -1/2 \\ 0 & 0 \end{bmatrix}$$

take $x_2 = -2$
(nice to cancel fraction)

$x_1 = \frac{1}{2} x_2 = -1$

So $\vec{v}_7 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

$\lambda = -1$

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 3/2 \\ 0 & 0 \end{bmatrix}$$

take $x_2 = 2$

$x_1 = -\frac{3}{2} x_2 = -3$

So $\vec{v}_{-1} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

Build: $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 7 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ -2 & 2 \end{bmatrix}^{-1}$

$= \begin{bmatrix} -1 & -3 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 7 & 0 \\ 0 & -1 \end{bmatrix} \cdot \frac{-1}{8}\begin{bmatrix} 2 & 3 \\ 2 & -1 \end{bmatrix}$

Ex Diagonalize $\begin{bmatrix} 12 & 8 \\ -7 & 11 \end{bmatrix}$.

Find λ: $\det\left(\begin{bmatrix} 12-\lambda & 8 \\ -7 & 11-\lambda \end{bmatrix}\right) = 0$

$$(12-\lambda)(11-\lambda) + 56 = 0$$

$$\lambda^2 - 23\lambda + 132 + 56 = 0$$

$$\lambda^2 - 23\lambda + 188 = 0$$

well $\lambda = \dfrac{23 \pm \sqrt{23^2 - 4(1)(188)}}{2}$

$$= \frac{23 \pm \sqrt{529 - 752}}{2}$$

$$= \frac{23 \pm \sqrt{\text{negative}}}{2}$$

This matrix has no real eigenvalues...

if you're willing to work with complex numbers

continue

if not, stop.

changed...

Ex Diagonalize $\begin{bmatrix} 12 & -8 \\ -7 & 3 \end{bmatrix}$

$\lambda = \dfrac{15 \pm \sqrt{225 - 4(-20)}}{2}$

$\lambda = \dfrac{15 \pm \sqrt{305}}{2}$

$\det\left(\begin{bmatrix} 12-\lambda & -8 \\ -7 & 3-\lambda \end{bmatrix}\right) = 0$

$$(12-\lambda)(3-\lambda) - 56 = 0$$

$$\lambda^2 - 15\lambda + 36 - 56 = 0$$

$$\lambda^2 - 15\lambda - \quad = 0 = 0$$

$$\lambda = \frac{15+\sqrt{305}}{2}$$

$$\begin{bmatrix} 12-\left(\frac{15+\sqrt{305}}{2}\right) & -8 \\ -7 & 3-\left(\frac{15+\sqrt{305}}{2}\right) \end{bmatrix}$$

RREF (div bottom row by -7)

$$\begin{bmatrix} 1 & -\frac{3}{7}+\frac{15+\sqrt{305}}{14} \\ 0 & 0 \end{bmatrix}$$

take $x_2 = 14$ to clear fractions

$$x_1 = -\left(\cancel{\quad}\right)x_2$$

$$= -\left(-6+15+\sqrt{305}\right) = -9-\sqrt{305}$$

$$\begin{bmatrix} -9-\sqrt{305} \\ 14 \end{bmatrix}$$

shortcut for 2nd e'vector:

flipped sign

$$\begin{bmatrix} -9+\sqrt{305} \\ 14 \end{bmatrix}$$

So
$$\begin{bmatrix} 12 & -8 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} -9-\sqrt{305} & -9+\sqrt{305} \\ 14 & 14 \end{bmatrix}\begin{bmatrix} \frac{15+\sqrt{305}}{2} & 0 \\ 0 & \frac{15-\sqrt{305}}{2} \end{bmatrix}\cdot P^{-1}$$

Ex If $A = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$. Diagonalize A.

It's already diagonal.

here $(6-\lambda)(6-\lambda) = 0$

$(6-\lambda)^2 = 0$

$\lambda = 6$ is the only e'value.

Ex If $A = \begin{bmatrix} 6 & 6 \\ 0 & 6 \end{bmatrix}$. Diagonalize A.

Find λ: $\det\left(\begin{bmatrix} 6-\lambda & 6 \\ 0 & 6-\lambda \end{bmatrix}\right) = 0$

$(6-\lambda)\cdot(6-\lambda) - 0 = 0$

$(6-\lambda)^2 = 0$

$\lambda = 6$ is the only eigenvalue...

Maybe using $\lambda = 6$ we could still find a basis ~~for~~ of $\mathbb{R}^2$ that had 2 eigenvectors...

$\lambda = 6$

Find eigenvectors...

$\begin{bmatrix} 0 & 6 \\ 0 & 0 \end{bmatrix}$

$\downarrow$

$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$x_2 = 0$

x_1 is free... take $x_1 = 1$.

Find e'vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Unable to find a second linearly indep. e'vector.

There is no basis of e'vectors $\Longrightarrow$ A can't be diagonalized.

Ex Diagonalize $\begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$. Find λ.

$$\det\left(\begin{bmatrix} 1-\lambda & 3 & 3 \\ -3 & -5-\lambda & -3 \\ 3 & 3 & 1-\lambda \end{bmatrix}\right) = 0$$

$$(1-\lambda)(-5-\lambda)(1-\lambda) - 27 - 27 + 9(1-\lambda) + 9(1-\lambda) - 9(-5-\lambda) = 0$$

$$(1-2\lambda+\lambda^2)(-5-\lambda) \quad -54 \quad + 18 - 18\lambda \quad + 45 + 9\lambda = 0$$

$$-5 + 9\lambda - 3\lambda^2 - \lambda^3 + 9 - 9\lambda = 0$$

$$-\lambda^3 - 3\lambda^2 + 4 = 0$$

$$\lambda^3 + 3\lambda^2 - 4 = 0 \quad \leftarrow \text{try to factor...}$$

easy to see $\lambda = 1$ is a root so $(\lambda - 1)$ is a factor

$$(\lambda-1)(\lambda^2 + 4\lambda + 4) = 0$$

$$(\lambda-1)(\lambda+2)^2 = 0$$

only 2 ~~[illegible]~~ e'vales...

So... $\lambda = 1$ and $\lambda = -2$

Need $\underline{\underline{3}}$ lin. indp. eigenvectors...

$$\begin{bmatrix} 0 & 3 & 3 \\ -3 & -6 & -3 \\ 3 & 3 & 0 \end{bmatrix}$$

$\downarrow$ RREF

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

take $x_3 = 1$

$x_2 = -x_3 = -1$

$x_1 = x_3 = 1$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} 3 & 3 & 3 \\ -3 & -3 & -3 \\ 3 & 3 & 3 \end{bmatrix}$$

$\downarrow$ RREF

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = -x_2 - x_3$

Gen sol. $\vec{x} = \begin{bmatrix} -x_2 - x_3 \\ x_2 \\ x_3 \end{bmatrix}$

$$x_2\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

!!!!

So...
$$\begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}^{-1}$$

Ex Diagonalize $\begin{bmatrix} 5 & -8 & 1 \\ 0 & 0 & 7 \\ 0 & 0 & 2 \end{bmatrix}$

Find λ:
$$(5-\lambda)(0-\lambda)(2-\lambda) = 0$$
$$-\lambda(5-\lambda)(2-\lambda)$$
$0 \quad 5 \quad 2$ are eigenvalues.

3 distinct e'values....

$\lambda = 0$
$$\begin{bmatrix} 5 & -8 & 1 \\ 0 & 0 & 7 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & -8/5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$
$x_3 = 0$

take $x_2 = 5$

$x_1 = \frac{8}{5} x_2 = 8$
$$\begin{bmatrix} 8 \\ 5 \\ 0 \end{bmatrix}$$

$\lambda = 5$
$$\begin{bmatrix} 0 & -8 & 1 \\ 0 & -5 & 7 \\ 0 & 0 & -3 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$
must be 0

take $x_1 = 1$
$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda = 2$
$$\begin{bmatrix} 3 & -8 & 1 \\ 0 & -2 & 7 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & -9 \\ 0 & 1 & -7/2 \\ 0 & 0 & 0 \end{bmatrix}$$
take $x_3 = 2$

$x_2 = \frac{7}{2} x_3 = 7$

$x_1 = 9x_3 = 18$
$$\begin{bmatrix} 18 \\ 7 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 1 & 18 \\ 5 & 0 & 7 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 8 & 1 & 18 \\ 5 & 0 & 7 \\ 0 & 0 & 2 \end{bmatrix}^{-1}$$

$\underline{\text{Ex}}$ Try to diagonalize $\begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$

Find λ: $\det \begin{bmatrix} 2-\lambda & 4 & 3 \\ -4 & -6-\lambda & -3 \\ 3 & 3 & 1-\lambda \end{bmatrix} = 0$

$\underbrace{(2-\lambda)(-6-\lambda)}(1-\lambda)\ \underbrace{-36\ -36}\ +9(2-\lambda)+16(1-\lambda)-9(-6-\lambda)=0$

$(-12+4\lambda+\lambda^2)(1-\lambda)\quad -72 + 18 - \cancel{9\lambda} + 16 - 16\lambda + 54 + \cancel{9\lambda} = 0$

$-12 + 16\lambda\ \underset{-3}{\cancel{\ \ \ }}\lambda^2 - \lambda^3 - 16\lambda + 16 = 0$

$-\lambda^3 - 3\lambda^2 + 4 = 0$

$\lambda^3 + 3\lambda^2 - 4 = 0$

$(\lambda-1)(\lambda+2)^2$

$\lambda = 1$ or $\lambda = -2$ are the two e'values

$\lambda = 1$

$\begin{bmatrix} 1 & 4 & 3 \\ -4 & -7 & -3 \\ 3 & 3 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$\downarrow$ RREF

$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & +1 \\ 0 & 0 & 0 \end{bmatrix}$

take $x_3 = 1$

$x_2 = -1$

$x_1 = 1$

$\lambda = -2$

$\begin{bmatrix} 4 & 4 & 3 \\ -4 & -4 & -3 \\ 3 & 3 & 3 \end{bmatrix}$

$\downarrow$ RREF

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

$x_3 = 0$

take $x_2 = 1$

$x_1 = -x_2 = -1$

Not enough lin. indep. e'vectors to build P!

So A is not diagonalizable!

Each eigenvalue has <u>multiplicity</u>

<u>algebraic multiplicity</u>

the power of $(\lambda - \lambda_0)$ in A's characteristic polynomial.

Last ex: $(\lambda-1)^1(\lambda+2)^2$

1 has alg mult. 1

-2 has alg mult 2

<u>geometric multiplicity</u>

is the dimension of the corresponding eigenspace.

<u>Fact:</u> algebraic mult $\geq$ geometric multiplicity ≥ 1