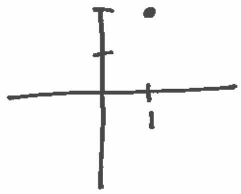
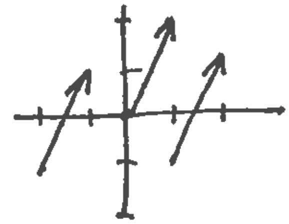
# 10.2 Vectors

$P = (1,2)$ is a point (location)



$\langle 1,2 \rangle$ is a vector (direction, & magnitude)



location we draw these is irrelevant.

$P = (a,b)$
$Q = (c,d)$

$\overrightarrow{PQ} = \langle c-a, d-b \rangle$
"Head − Tail"

To refer to a vector's magnitude

$\|\vec{v}\|$ is $\vec{v}$'s magnitude

$\|\langle a,b \rangle\|$ = distance from $(0,0)$ to $(a,b)$

$= \sqrt{a^2 + b^2}$

$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \cdots v_n^2}$

in higher dimensions

Writing $\langle 1,2 \rangle$ is using component form for a vector

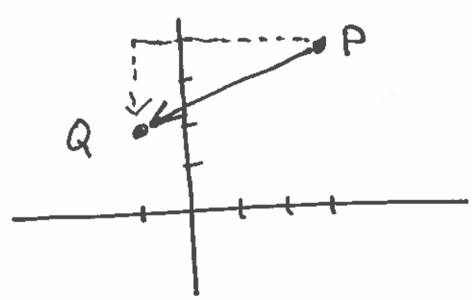
list "component"s,

Traditional variable: $\vec{v} = \langle 1,2 \rangle$

Given points $P$, $Q$, we could imagine vector from $P$ to $Q$, $\overrightarrow{PQ}$

Ex $P = (3,4)$
$Q = (-1,2)$

$\overrightarrow{PQ}$




Find $\overrightarrow{PQ}$'s component form:
$\langle -1-3, 2-4 \rangle = \langle -4, -2 \rangle$

Ex $\|\langle 7,3\rangle\| = \sqrt{49+9} = \sqrt{58}$

Ex $\|\overrightarrow{PQ}\|$ where $P = (-1,3)$ ...
$Q = (5,2)$

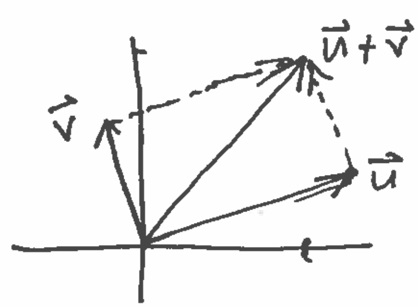
well, $\overrightarrow{PQ} = \langle 6,-1\rangle$

$= \sqrt{36+1} = \sqrt{37}$

Facts If $\vec{u} = \langle u_1, u_2\rangle$
$\vec{v} = \langle v_1, v_2\rangle$

We can add: $\vec{u} + \vec{v} = \underbrace{\langle u_1+v_1, u_2+v_2\rangle}_{\text{result is another vector}}$

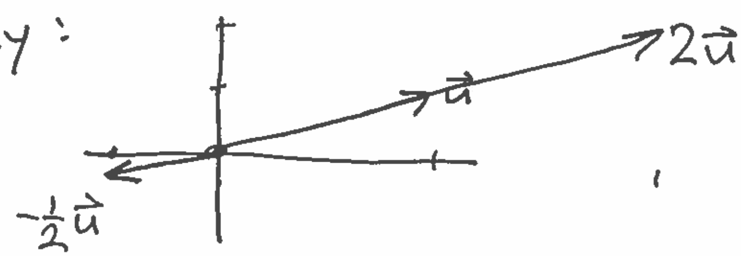
Visually: Makes parallelograms




We can multiply by a scalar:

$c\cdot\vec{u} = \langle cu_1, cu_2\rangle$

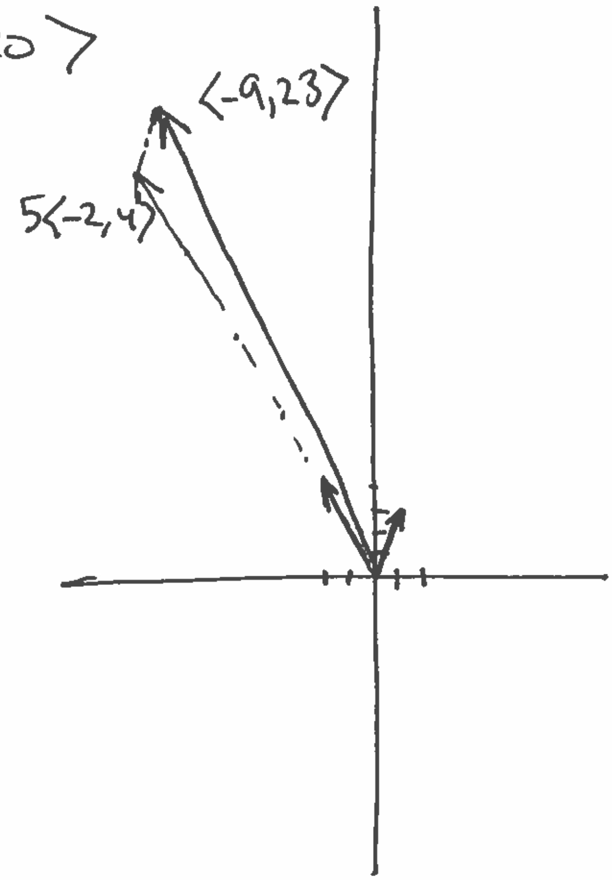
Visually:



Ex $\langle 1,3\rangle + 5\langle -2,4\rangle$

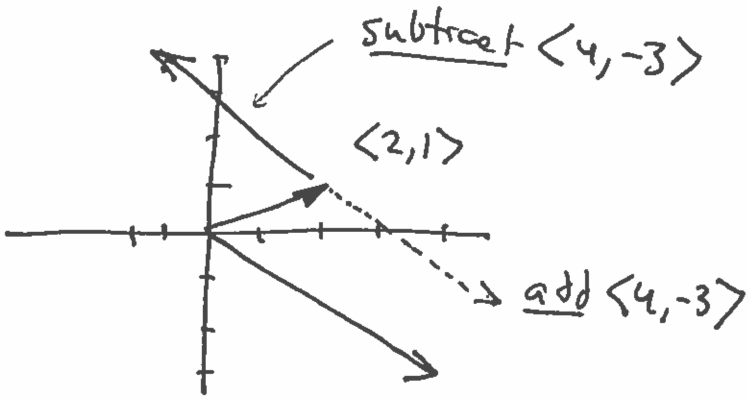
$= \langle 1,3\rangle + \langle -10,20\rangle$

$= \langle -9,23\rangle$




Ex $\langle 2,1\rangle - \langle 4,-3\rangle$

$= \langle -2,4\rangle$



## More Facts

$$\vec{u} + \vec{v} = \vec{v} + \vec{u}$$

$$(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$$

$$\vec{v} + \vec{0} = \vec{v}$$

$\uparrow$

Zero vector $\langle 0,0 \rangle$ in 2D

$\langle 0,0,0 \rangle$ in 3D

$\vdots$

$$(cd)\vec{u} = c(d\vec{u})$$

$$c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$$

$$(c+d)\vec{u} = c\vec{u} + d\vec{u}$$

$$0 \cdot \vec{v} = \vec{0}$$

$$\| c \cdot \vec{v} \| = |c| \cdot \|\vec{v}\|$$

$$\|\vec{v}\| = 0 \iff \vec{v} = \vec{0}$$

(if $\vec{v} \neq \vec{0}$, $\|\vec{v}\| \neq 0$, so it would be ok to divide by $\|\vec{v}\|$.)

A <u>unit vector</u> is a vector $\vec{v}$ with $\|\vec{v}\| = 1$

Note: If $\vec{v}$ is nonzero $\dfrac{\vec{v}}{\|\vec{v}\|}$ is a <u>vector</u> $\left(\dfrac{1}{\|\vec{v}\|}\vec{v}\right)$.

$$\left\|\frac{\vec{v}}{\|\vec{v}\|}\right\| = \left\|\frac{1}{\|\vec{v}\|}\cdot\vec{v}\right\| = \left|\frac{1}{\|\vec{v}\|}\right|\cdot\|\vec{v}\|$$

$$= \frac{1}{\|\vec{v}\|}\|\vec{v}\| = 1$$

So $\dfrac{\vec{v}}{\|\vec{v}\|}$ is a unit vector in same direction as $\vec{v}$.

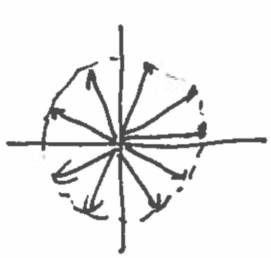
<u>Ex</u> $\langle 1, 2, 2\rangle$. Find a unit vector in same direction.

$$\|\langle 1, 2, 2\rangle\|$$

$$= \sqrt{1+4+4} = \sqrt{9} = 3$$

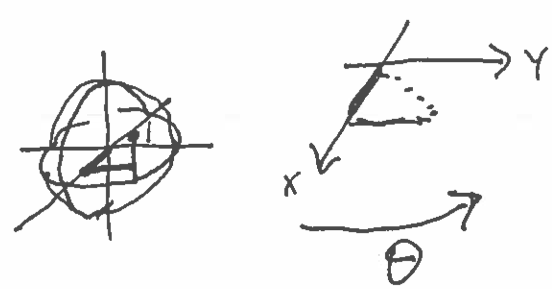
So $\dfrac{\langle 1,2,2\rangle}{3} = \underline{\left\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right\rangle}$.

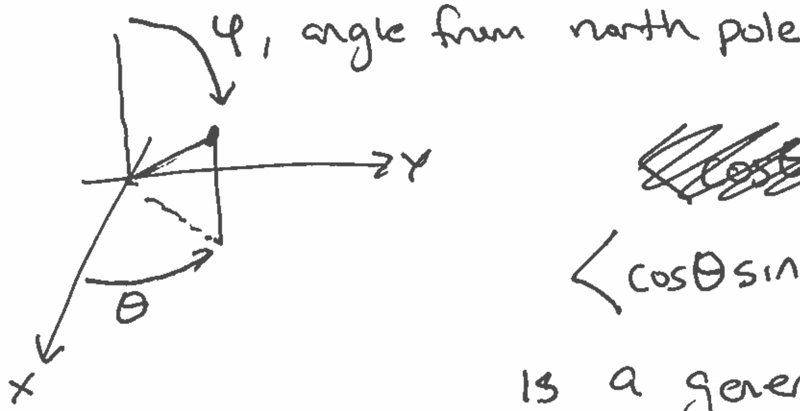
Unit vectors in 2D ($\mathbb{R}^2$)



$$\underbrace{\langle \cos\theta, \sin\theta\rangle}_{\text{all unit vectors in this form.}}$$

Unit vectors in 3D ($\mathbb{R}^3$)



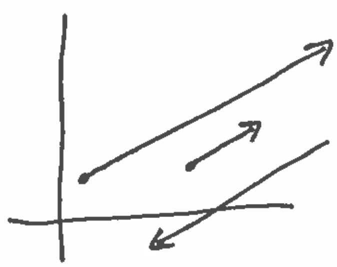



$\langle \cos\theta \sin\varphi, \sin\theta \sin\varphi, \cos\varphi \rangle$

is a generic unit vector.

(spherical coordinates, ....
if you throw in a radius
variable $\rho$.)

## Vectors can be parallel



Yes: one vector is a nonzero multiple of the other: $\vec{u} \parallel \vec{v} \iff \vec{u} = c \cdot \vec{v}$

"is parallel"

non zero

Ex $\langle 15, 20, -25 \rangle$
$\langle -21, -28, 35 \rangle$

Are these parallel?

Convert to unit vectors:

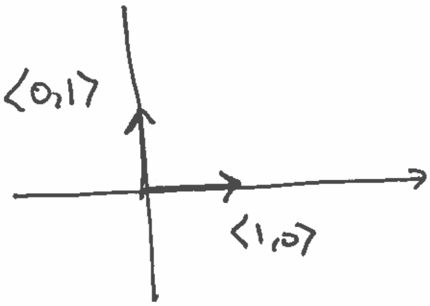
$\left\langle \frac{3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$

$\left\langle \frac{-3}{5\sqrt{2}}, \frac{-4}{5\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

Yes! $\vec{u} \parallel \vec{v}$
if and only if

$$\frac{\vec{u}}{\|\vec{u}\|} = \frac{\vec{v}}{\|\vec{v}\|} \quad \text{or} \quad \frac{\vec{u}}{\|\vec{u}\|} = -\frac{\vec{v}}{\|\vec{v}\|}$$

# Standard unit vectors



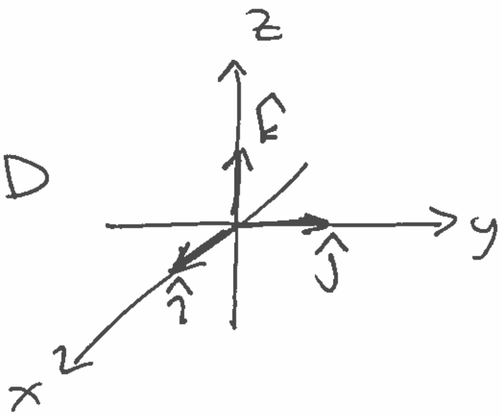

$\hat{\imath} = \langle 1,0\rangle$

$\hat{\jmath} = \langle 0,1\rangle$

So $\langle 2,3\rangle = 2\langle 1,0\rangle + 3\langle 0,1\rangle$

$\langle 2,3\rangle = \underbrace{2\hat{\imath} + 3\hat{\jmath}}_{\text{standard unit vector form.}}$

In 3D




$\hat{\imath} = \langle 1,0,0\rangle$

$\hat{\jmath} = \langle 0,1,0\rangle$

$\hat{k} = \langle 0,0,1\rangle$

$3\hat{\imath} - \hat{\jmath} + 5\hat{k} = \langle 3,-1,5\rangle$