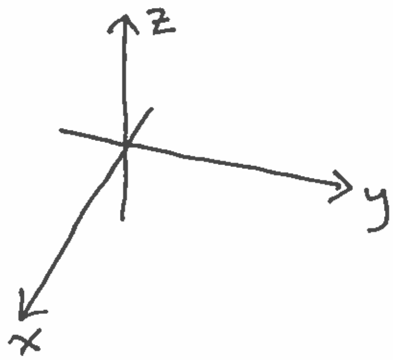
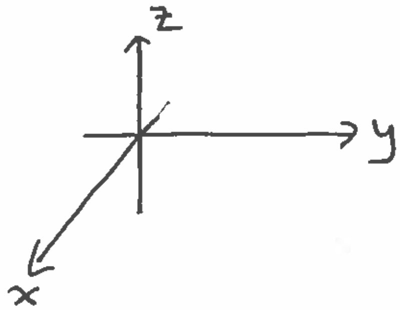
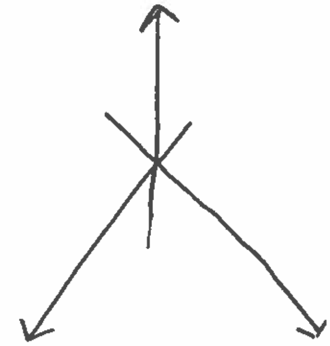
10.1 Coordinates in Space

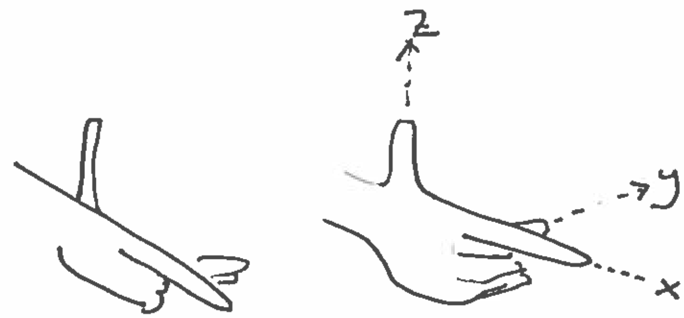


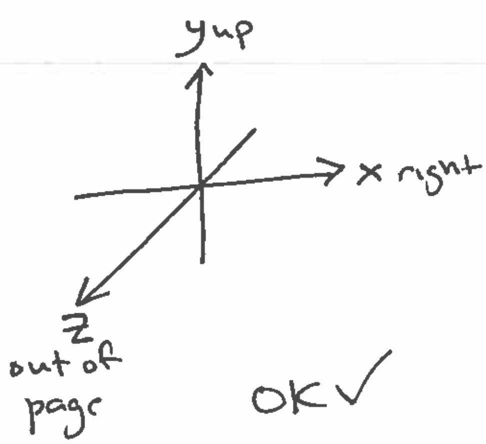




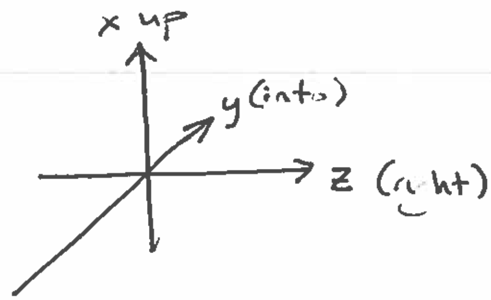
Right-Hand Rule:

However you draw, respect this rule.



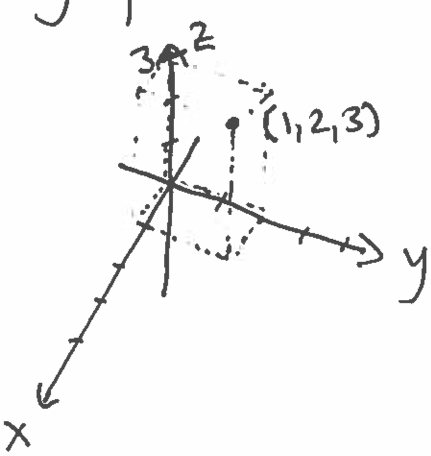


OK ✓

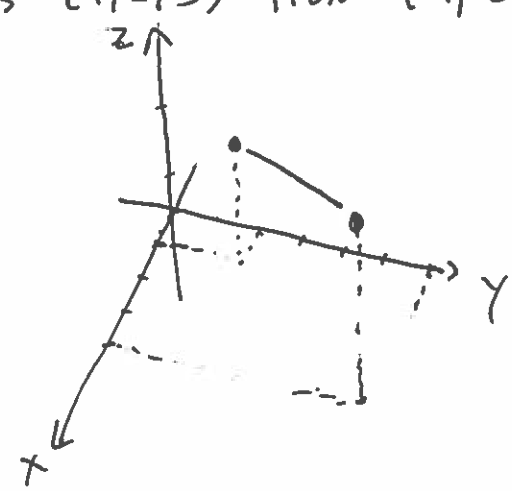


Not OK ✗

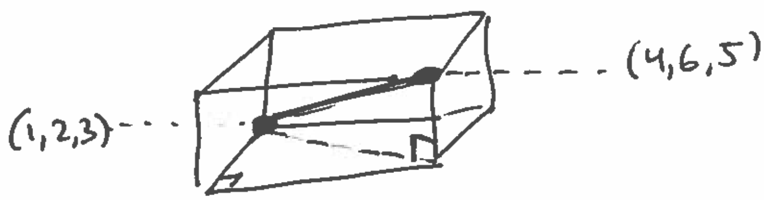
Plotting points: (1,2,3).

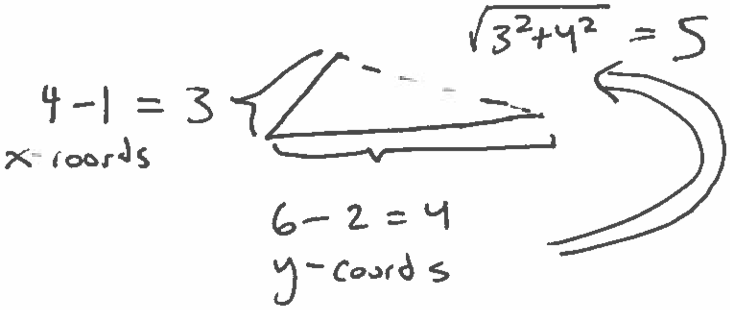


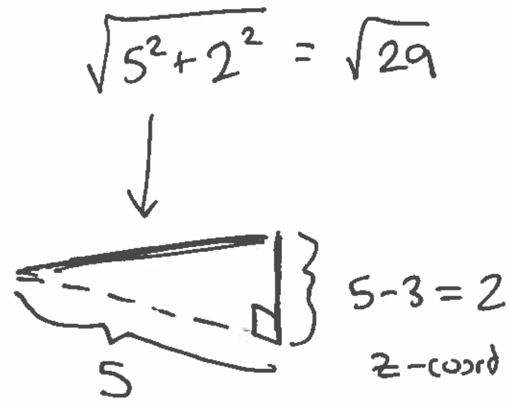
Distance: How far is (1,2,3) from (4,6,5)?



Two points as opposite ~~ea~~ corners of a box







So $d = \sqrt{\left(\sqrt{3^2+4^2}\right)^2 + 2^2}$

$= \sqrt{3^2+4^2+2^2}$

$d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$

<u>Ex</u> $(1,-1,4)$ & $(0,2,8)$.

Find distance.

$d = \sqrt{1^2+3^2+4^2}$

$= \sqrt{26}$

Basic shapes in 3D

<u>Spheres</u> the collection of all points some <u>fixed distance</u> (radius), from some special center.

distance between center and generic surface

some specific #

$\Longrightarrow d = r$

$\sqrt{\Delta x^2+\Delta y^2+\Delta z^2} = r$

$\Delta x^2+\Delta y^2+\Delta z^2 = r^2$

If center is (a,b,c) ...

$\underbrace{(x-a)^2+(y-b)^2+(z-c)^2=r^2}_{\text{Equation for a sphere.}}$

Ex I need a sphere of radius 8, centered at $(2,-4,5)$. $\Longrightarrow (x-2)^2+(y+4)^2+(z-5)^2 = 64$

Ex $\underbrace{x^2+2x}$ $+y^2-4y+z^2-6z = 2$

By the way, this makes a sphere.
Where is center? What is radius?

$\downarrow$

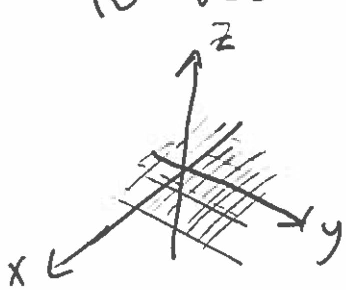
$(x+1)^2$
but no!!!

$$\underbrace{x^2+2x+1}_{(x+1)^2} + y^2-4y+4 + z^2-6z+9 = 2+1+4+9$$

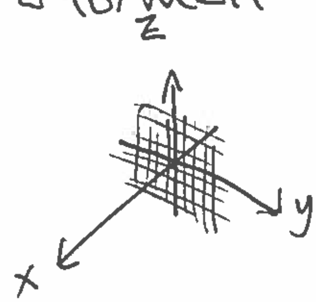
$$(x+1)^2 + (y-2)^2 + (z-3)^2 = 16$$

$\Longrightarrow$ Center: $(-1,2,3)$ Radius $= 4$

Planes very basic shape but not so easy to describe with a formula yet...



xy-plane: $z=0$

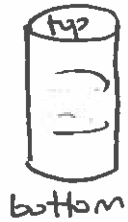


yz-plane: $x=0$

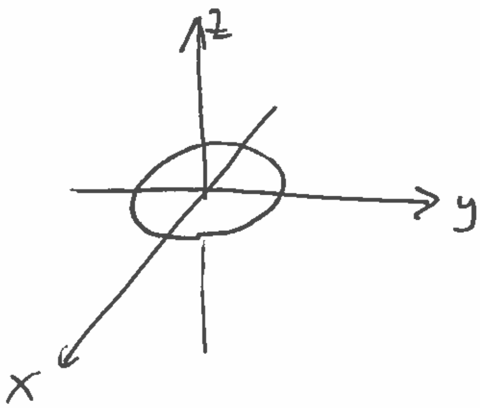
And xz-plane $y=0$

"Cylinders"

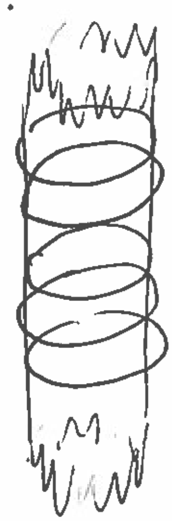
Cylinder is more general than this



When we take some 2D plane curve embedded in one of the cardinal planes...



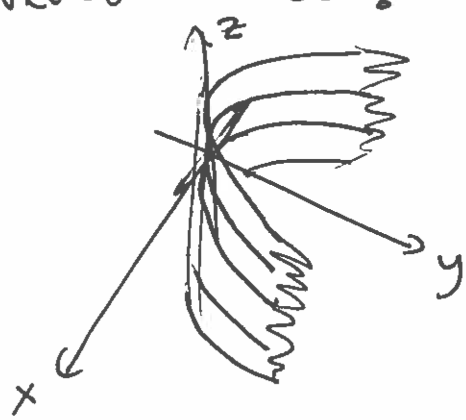
and then take all points where 3rd coordinate can be anything.



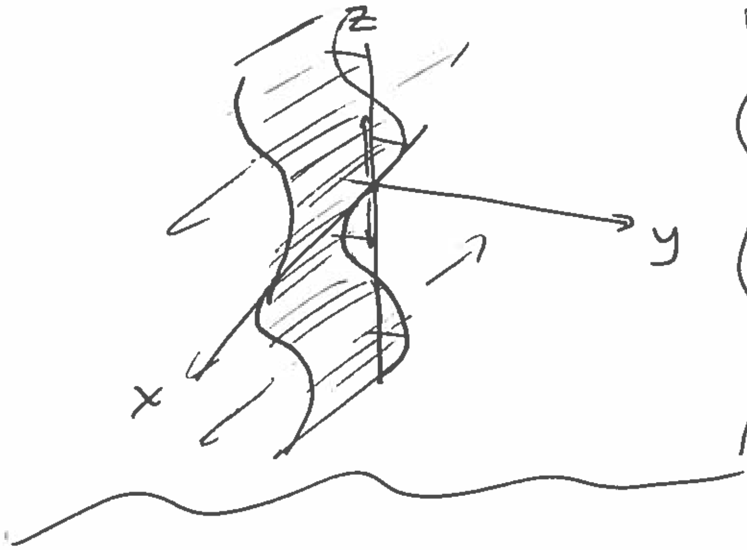
the ~~basic~~ 2D shape need not be a circle

Ex $y = x^2$

viewed in 3D!

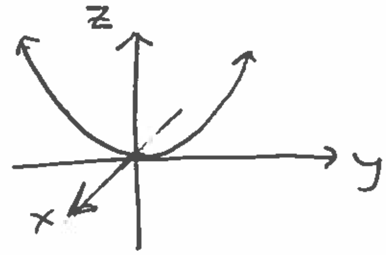


Ex Does $\underbrace{y = \sin(z)}_{\text{missing } x}$ make a cylinder?

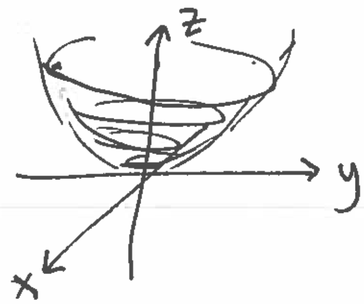


Surfaces of Revolution

Ex $\quad z = y^2$



and rotate about z-axis:



what is the equation for this surface?

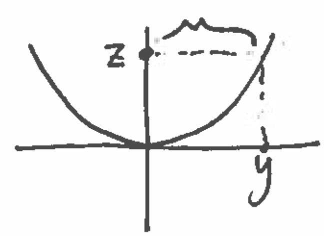
We have circular cross-sections in which 2 directions: x & y

depends on z

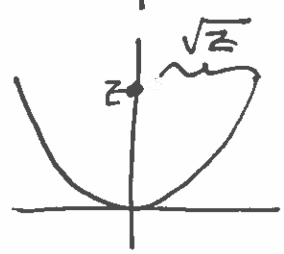
$$x^2 + y^2 = (\text{radius})^2$$

y becomes a radius!

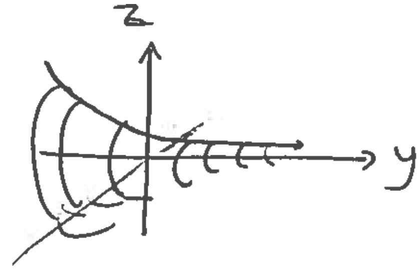
$$\boxed{x^2 + y^2 = (\sqrt{z})^2}$$



$$z = y^2$$
$$\Rightarrow y = \sqrt{z}$$



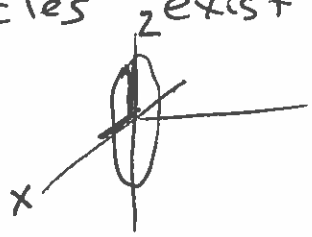
Ex $z = e^{-y}$



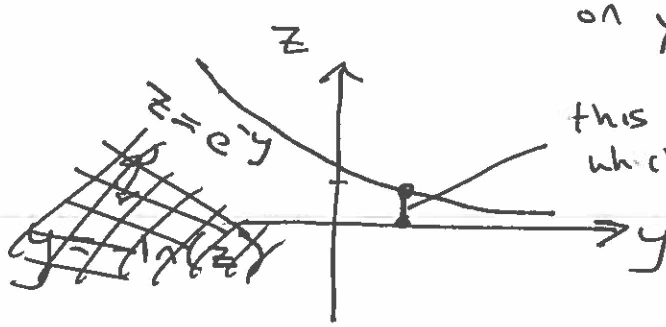
Rotate about y-axis.

What is the equation for this "horn".

Circles exist in x, z directions



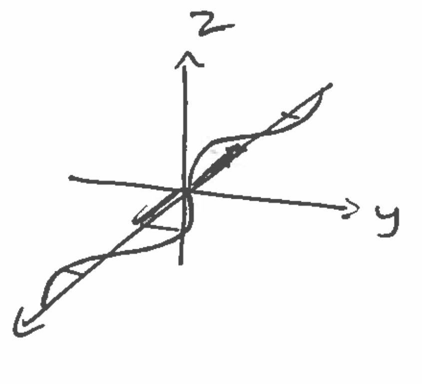
$$x^2 + z^2 = \left(\begin{array}{c}\text{radius} \\ \text{that depends} \\ \text{on } y\end{array}\right)^2$$



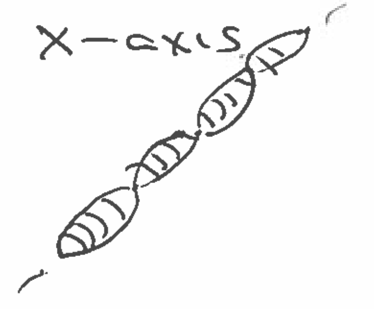
this is z which $\underline{\underline{\text{is}}}$ e^{-y}

$$\text{So } x^2 + z^2 = (e^{-y})^2.$$

Ex $y = \sin(x)$



Rotate about x-axis

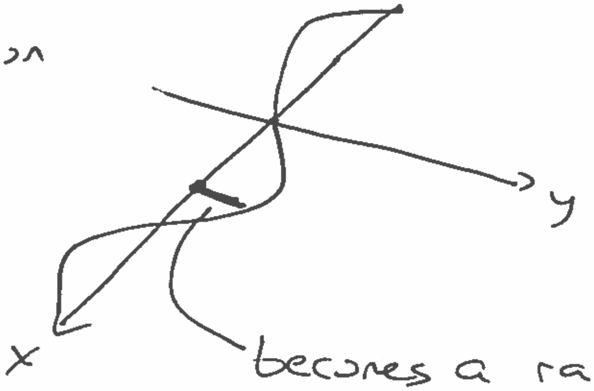


Circles in y & z dir

$y^2 + z^2 = (\text{radius depends on } x)$

Find its equation.

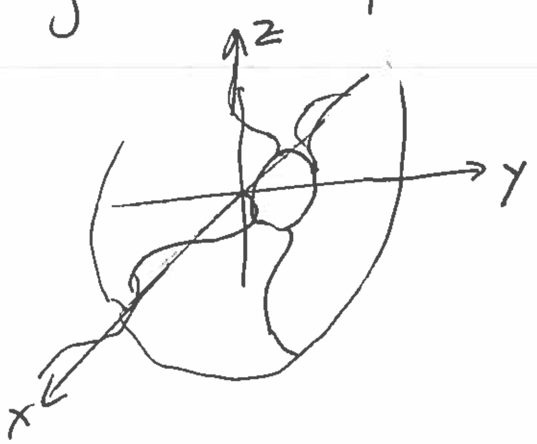
2D version



becomes a radius: $y = \sin(x)$

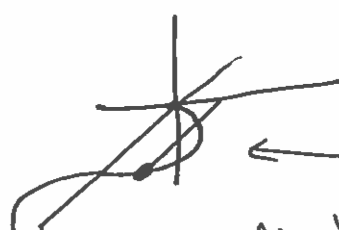
So $y^2 + z^2 = (\sin x)^2$

Ex $y = \sin(x)$, rotate around y-axis



$$x^2 + z^2 = \begin{pmatrix}\text{radius}\\ \text{in terms}\\ \text{of } y\end{pmatrix}^2$$

2D picture



← x-value

Need: Solve for x in terms of y

$$x^2 + z^2 = (\text{radius})^2$$

$$\sqrt{x^2 + z^2} = |\text{radius}|$$

$$\pm\sqrt{x^2 + z^2} = \text{radius}$$

to free radius!

(the x-value from this picture)

Can't do this because $y = \sin(x)$ has noninvertible function...

$$\pm\sqrt{x^2 + z^2} = \underset{\text{from picture}}{x} \Longrightarrow \sin\left(\pm\sqrt{x^2+z^2}\right) = \sin\left(\underset{\text{from picture}}{x}\right)$$

$$\sin\left(\pm\sqrt{x^2+z^2}\right) = y$$

Quadric Surfaces

$$A \cdot x^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz$$
$$+ Gx + Hy + Iz + J = 0$$

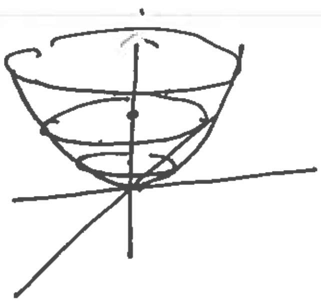
Makes one of 5 or 6 shapes....

Special Cases

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

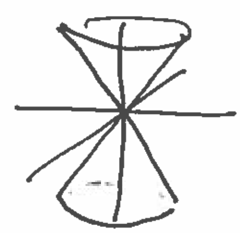
Fix z-value, d.

$$d = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$
$$1 = \frac{x^2}{a^2 d} + \frac{y^2}{b^2 d}$$



- z, singled out $\Rightarrow$ tells you central axis.
- $\frac{x^2}{a^2} + \frac{y^2}{b^2} \Longrightarrow$ cross-sections are ellipses.
- $\boxed{\text{elliptic paraboloid}}$

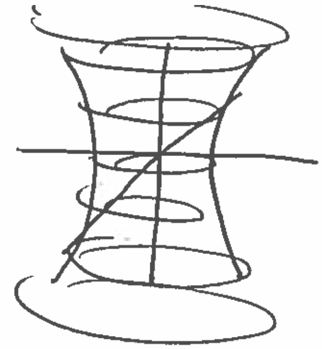
$$z^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} \Longrightarrow \text{elliptic cone}$$



z singled out $\Longrightarrow$ central axis

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

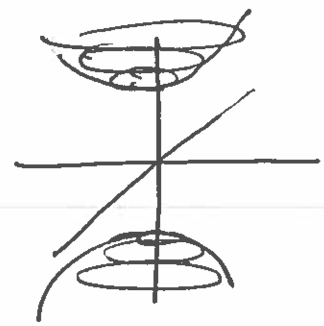
z singled out $\Longrightarrow$ central axis in z-direction



Hyberboloid of One Sheet

$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

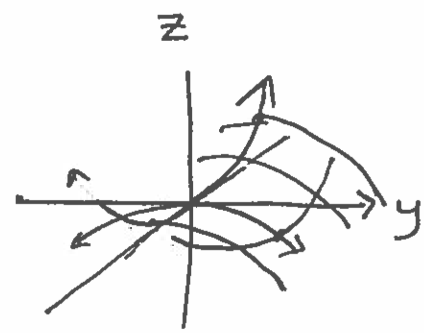
z singled out $\Longrightarrow$ central axis in z-dir



Hyperboloid of two sheets

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

z $\rightarrow$ single out $\rightarrow$ central axis



Hyperbolic paraboloid

zy-plane $\Longrightarrow x = 0$

$\Longrightarrow z = -\frac{y^2}{b}$

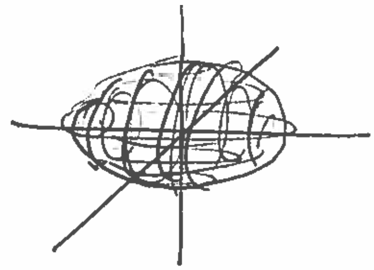
$\Longrightarrow$ downward parabola

xz-plane $\Longrightarrow y = 0$

$\Longrightarrow z = \frac{x^2}{a^2}$

$\Longrightarrow$ upward parabola

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



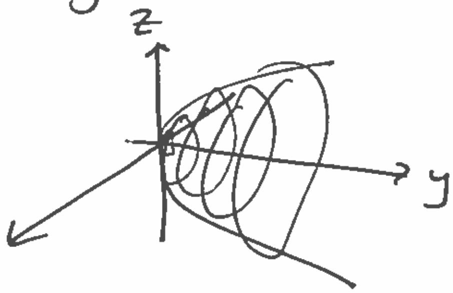
$\boxed{\text{Ellipsoid}}$

a, b, c are
x-, y-, z-radii
respectively.

Ex Sketch $y = \frac{x^2}{4} + \frac{z^2}{16}$

$\boxed{\text{Elliptic Paraboloid}}$

y central axis



Sketch:

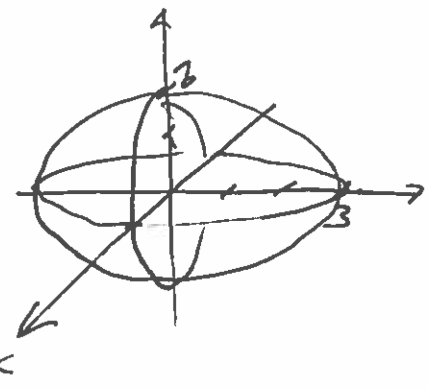
Ex $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$

$\boxed{\text{Ellipsoid}}$

x-radius 1

y-radius 3

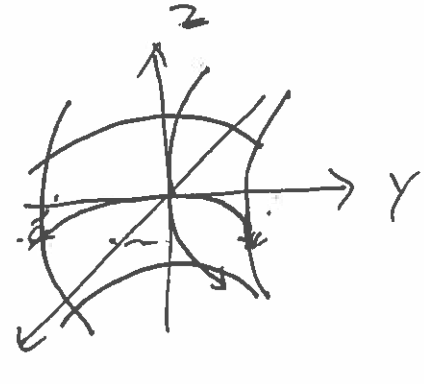
z-radius 2



Ex $x = y^2 - z^2$

$\boxed{\text{Hyperbolic Paraboloid}}$

x is central



xy-plane
$z=0$
$x=y^2$
pos opening
parabola

xz plane
$y=0$
$x=-z^2$