KEY

Math 95 Final Exam Review

1. Evaluating Functions and Determining their Domain and Range

The **domain** of a function is the set of all possible inputs, which are typically x-values. The **range** of a function is the set of all possible outputs, which are typically y-values.

When determining the domain of a function based on the formula:

- Exclude any numbers that cause **division by zero**.
- Exclude any numbers that cause **the square root of a negative number** to occur.

Example 1. Algebraically determine the domain of the following functions. State each domain using set-builder notation AND interval notation. Then find $f(0)$ and $f(-2)$ for each function.

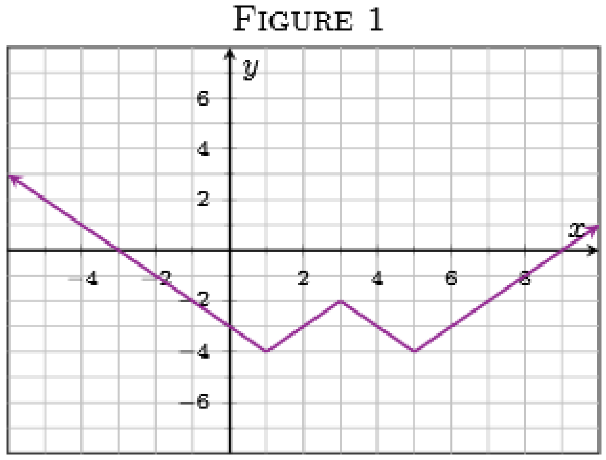
Function	Domain (SB Notation)	Domain (Interval Notation)	$f(0)$	$f(-2)$
$f(x) = -5x + 7$	$\{x \mid x \text{ is a real number}\}$	$(-\infty, \infty)$	7	17
$f(x) = \sqrt{-2x+3}$	$\{x \mid x \leq \frac{3}{2}\}$	$(-\infty, 3/2]$	$\sqrt{3}$	$\sqrt{7}$
$f(x) = -3(x-1)^2 - 4$	$\{x \mid x \text{ is a real number}\}$	$(-\infty, \infty)$	-7	-31
$f(x) = \frac{4x+1}{3x-6}$	$\{x \mid x \neq 2\}$	$(-\infty, 2) \cup (2, \infty)$	$-\frac{1}{6}$	$\frac{7}{12}$

$-2x+3 \geq 0$
$3 \geq 2x$
$\frac{3}{2} \geq x$
$x \leq \frac{3}{2}$

$3x-6 \neq 0$
$3x \neq 6$
$x \neq 2$

Example 2. State the domain and range of each function below using interval notation.

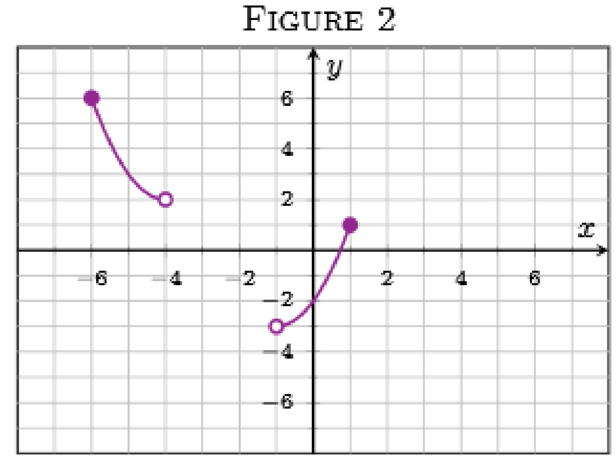
FIGURE 1



Domain: $(-\infty, \infty)$

Range: $[-4, \infty)$

FIGURE 2



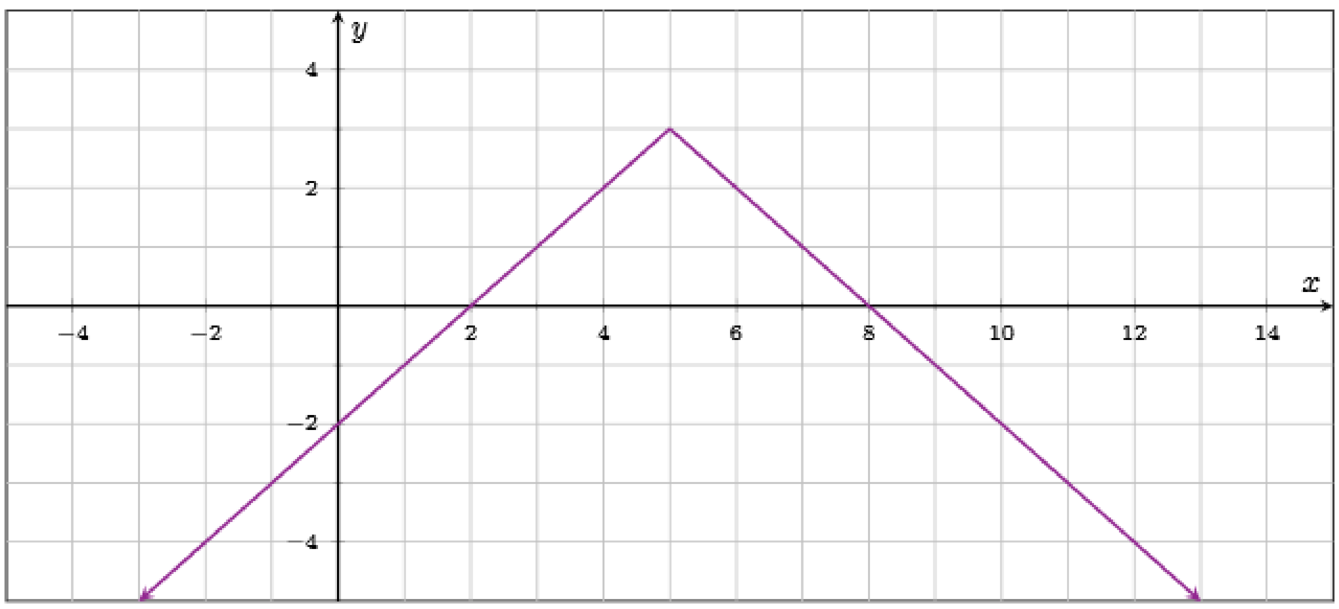
Domain: $[-6, -4) \cup (-1, 1]$

Range: $(-3, 1] \cup (2, 6]$

2. Evaluating, Solving, and Determining Domain and Range Using the Graph of a Function

Example 3. Use the graph of $y = h(x)$ in Figure 3 to answer the following. State all solutions to equations in a solution set.

Figure 3



(a) State the domain of h.

$(-\infty, \infty)$

(b) State the range of h.

$(-\infty, 3]$

(c) Evaluate $h(0)$.

$h(0) = -2$

(d) Solve $h(x) = 0$.

$x = 2$ or $x = 8$

Solution Set: $\{2, 8\}$

(e) State the following:

x-intercept(s): $(2,0)$ and $(8,0)$

y-intercept: $(0,-2)$

(f) Evaluate $h(1)$.

$h(1) = -1$

(g) Solve $h(x) = 1$.

$x = 3$ or $x = 7$

Solution Set: $\{3, 7\}$

(h) Solve $h(x) = 3$.

$x = 5$

Solution Set: $\{5\}$

(i) Solve $h(x) \leq 2$.

$x \leq 4$ or $x \geq 6$

Solution Set: $(-\infty, 4] \cup [6, \infty)$

(j) Solve $h(x) > -3$.

$-1 < x < 11$

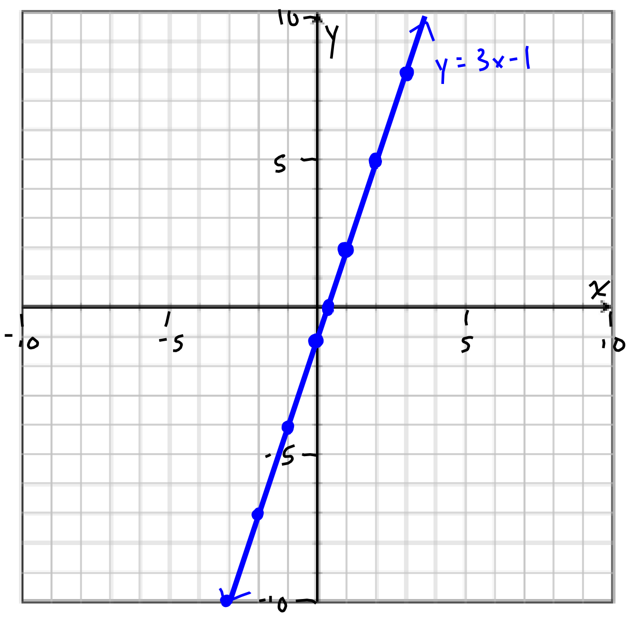
Solution Set: $(-1, 11)$

3. Sketching Graphs of Functions By Hand

Example 4. Sketch the graph of $y = f(x)$ for each of the following functions **WITHOUT** a calculator. Clearly label any x-intercepts and y-intercepts.

(a) $f(x) = 3x - 1$

Figure 4

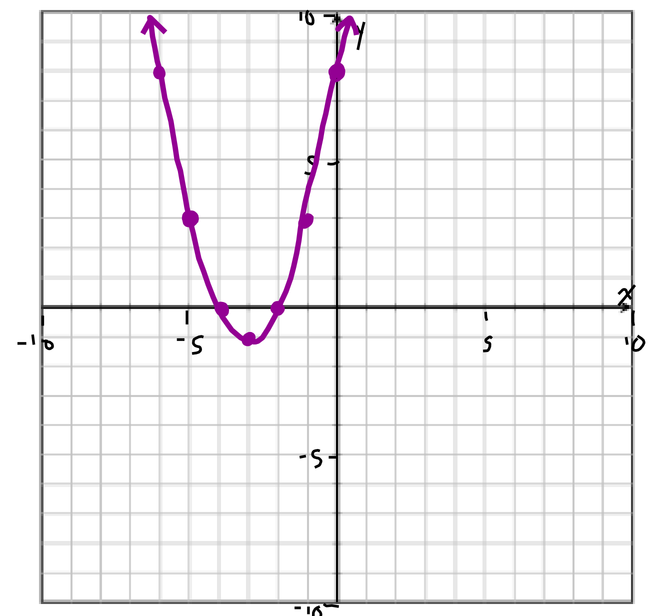


x-intercept: $(\frac{1}{3}, 0)$

y-intercept: $(0, -1)$

(c) $f(x) = (x+3)^2 - 1$

Figure 6



vertex: (-3, -1)

x	y
-5	3
-4	0
-3	-1
-2	0
-1	3
0	8

x-intercepts: (-4,0) and (-2,0)

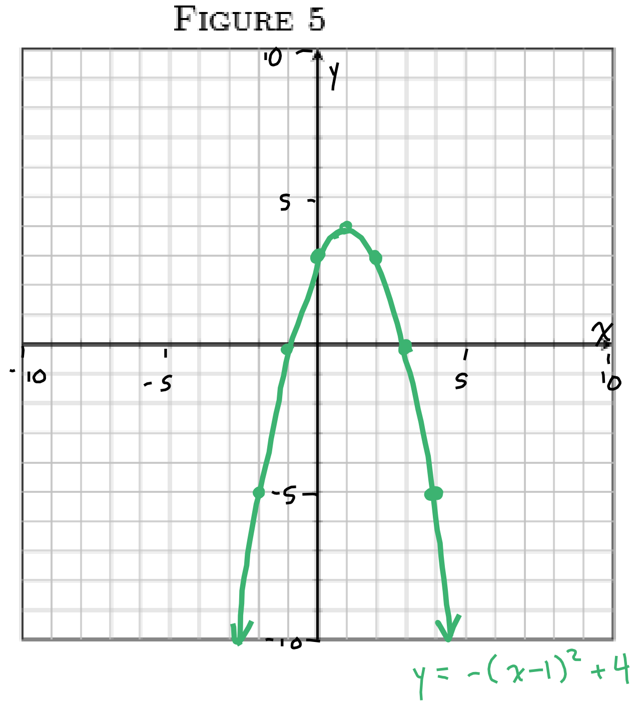
y-intercept: (0, 8)

(b) $f(x) = -(x-1)^2 + 4$

vertex: (1,4)

x	y
-2	-5
-1	0
0	3
1	4
2	3
3	0
4	-5

Figure 5

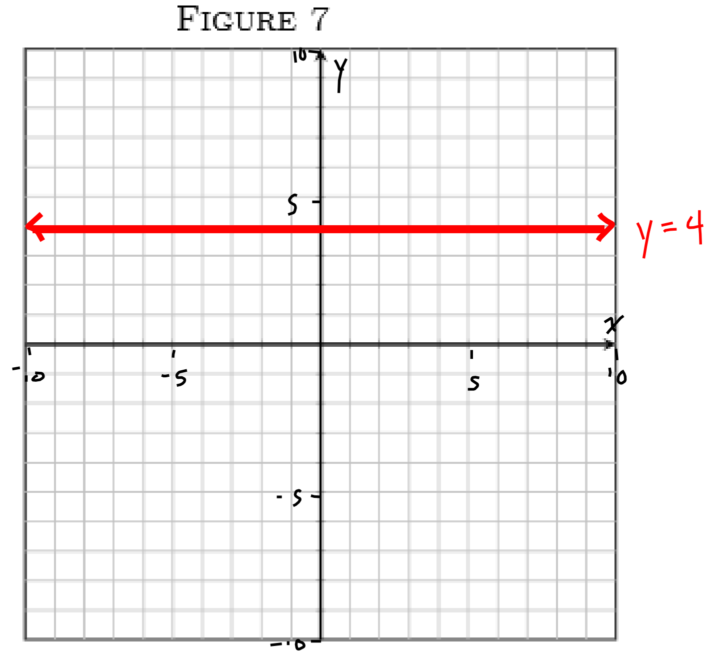


x-intercepts: (-1,0) and (3,0)

y-intercept: (0,3)

(d) $f(x) = 4$

Figure 7



x-intercept: none

y-intercept: (0,4)

4. Finding the Formula of a Linear Function

Example 5. Suppose that $y = g(x)$ is a linear function and that $g(-2) = 11$ and $g(5) = -3$. Find the algebraic formula for $g(x)$. Points: $(-2, 11)$ and $(5, -3)$

$m = \frac{-3-11}{5-(-2)}$

$= \frac{-14}{7}$

$= -2$

$y = -2x + b$

$11 = -2(-2) + b$

$11 = 4 + b$

$7 = b$

Formula: $g(x) = -2x + 7$

Example 6. Suppose that $y = h(t)$ is a linear function and that $h(4) = 6$ and $h(10) = 9$. Find the algebraic formula for $h(t)$. Points: $(4, 6)$ and $(10, 9)$

$m = \frac{9-6}{10-4}$

$= \frac{3}{6}$

$= \frac{1}{2}$

$y = \frac{1}{2}t + b$

$6 = \frac{1}{4}(4) + b$

$6 = 2 + b$

$4 = b$

Formula: $h(t) = \frac{1}{2}t + 4$

5. Completing the Square

Example 7. Write the quadratic equation $y = x^2 + 12x + 5$ in vertex form by completing the square. Identify the vertex.

$y = x^2 + 12x + 5$

$y = x^2 + 12x + 36 - 36 + 5$

$y = (x^2 + 12x + 36) - 36 + 5$

$y = (x+6)^2 - 36 + 5$

$y = (x+6)^2 - 31$

$b = 12$

$\frac{b}{2} = 6$

$\left(\frac{b}{2}\right)^2 = 36$

Vertex: $(-6, -31)$

Example 8. Solve quadratic equation $x^2 - 6x + 1 = -10$ using completing the square. Clearly state all real and complex solutions in a solution set.

$x^2 - 6x + 1 = -10$

$x^2 - 6x + 9 - 9 + 1 = -10$

$(x^2 - 6x + 9) - 9 + 1 = -10$

$(x-3)^2 - 8 = -10$

$(x-3)^2 = -2$

Continued:

$x - 3 = \pm\sqrt{-2}$

$x - 3 = \pm\sqrt{-1}\sqrt{2}$

$x - 3 = \pm i\sqrt{2}$

$x = 3 \pm i\sqrt{2}$

Solution set: $\{3 + i\sqrt{2}, 3 - i\sqrt{2}\}$

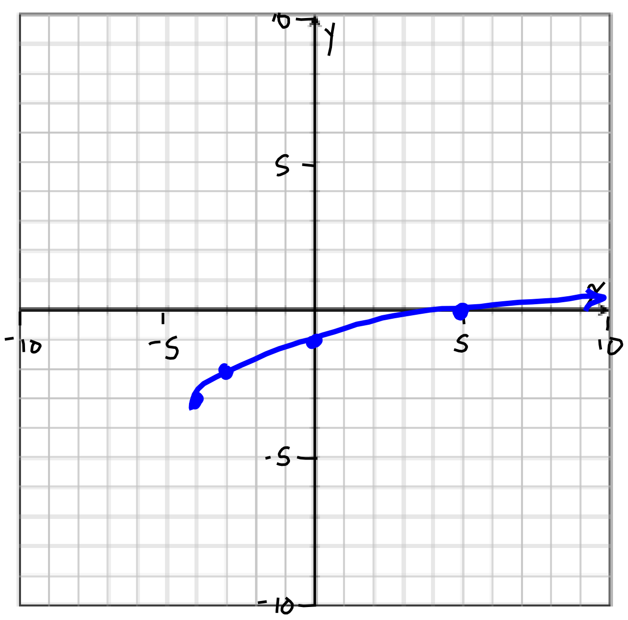
6. Graphing Functions and Solving Equations with your Graphing Calculator

Example 9. Sketch the graph of $y = f(x)$ for each of the following functions **WITH** the help of your calculator. Clearly label any x-intercepts and y-intercepts.

(a) $f(x) = \sqrt{x+4} - 3$

x	y
-4	-3
-3	-2
0	-1
5	0
10	≈0.74

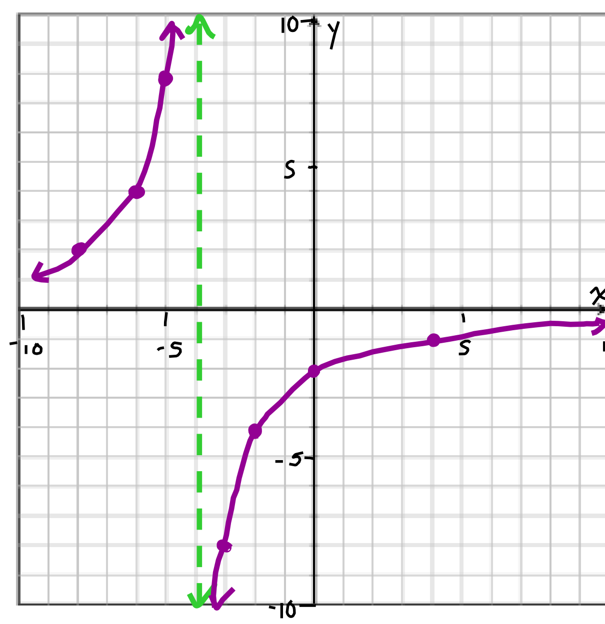
Figure 8



x-intercept: (5,0) y-intercept: (0,-1)

(b) $f(x) = \frac{-8}{x+4}$

Figure 9



x = -4

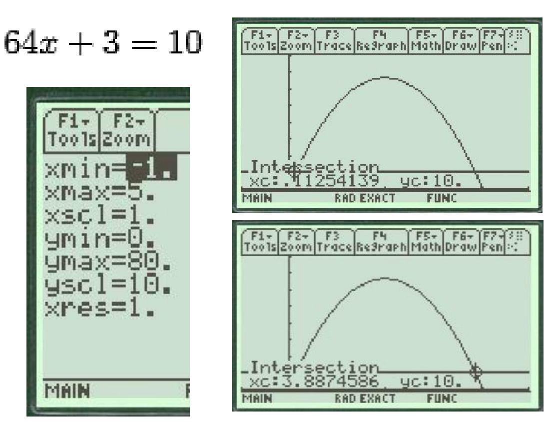
x	y
-8	2
-6	4
-5	8
-4	undef.
-3	-8
-2	-4
0	-2
4	-1
10	-0.5

y-intercept: (0,-2)

Example 10. **Graphically** solve the following equations and inequalities using your graphing calculator. Clearly state each solution set. Round all solutions accurate to 3 decimal places.

(a) $-16x^2 + 64x + 3 = 10$

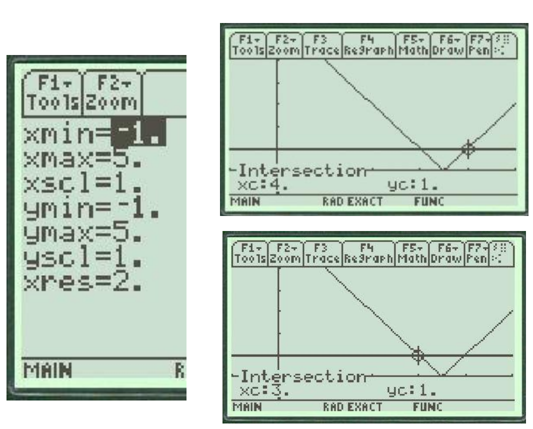
$\{x \mid x \approx 0.112, 3.887\}$



(b) $|-2x + 7| > 1$

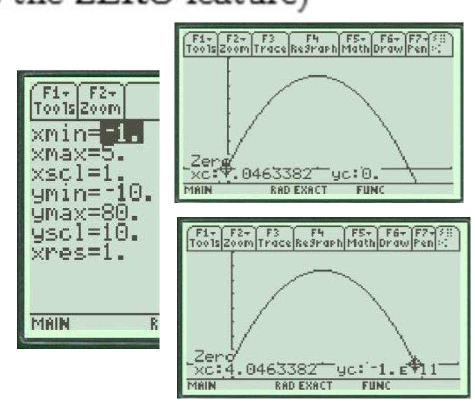
Solution Set:

$(-\infty, 3) \cup (4, \infty)$



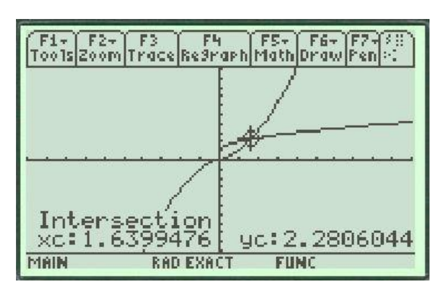
(c) $-16x^2 + 64x + 3 = 0$ (Use the ZERO feature)

$\{x \mid x \approx 0.046, 4.046\}$



(d) $x^{5/3} = x^{1/2} + 1$

$\{x \mid x \approx 1.640\}$



7. Solving Equations Algebraically

Example 11. **Algebraically** solve the following equations and inequalities. Clearly state each solution set.

(a) $-8 < -4x + 12 \leq 16$

$$-8 < -4x + 12 \leq 16$$

$$-8 - 12 < -4x + 12 - 12 \leq 16 - 12$$

$$-20 < -4x \leq 4$$

$$\frac{-20}{-4} > \frac{-4x}{-4} \geq \frac{4}{-4}$$

$$5 > x \geq -1$$

$$-1 \leq x < 5$$

Solution set: $\{x \mid -1 \leq x < 5\}$

OR $[-1, 5)$

(b) $\sqrt{2x + 18} + 3 = x$

$$\sqrt{2x+18} + 3 = x$$

$$\sqrt{2x+18} = x - 3$$

$$\left(\sqrt{2x+18}\right)^2 = (x-3)^2$$

$$2x + 18 = x^2 - 6x + 9$$

$$0 = x^2 - 8x - 9$$

$$0 = (x-9)(x+1)$$

$x - 9 = 0$ OR $x + 1 = 0$

$x = 9$ $\quad$ $x = -1$

Check: $x = 9$

$$\sqrt{2(9)+18} + 3 \stackrel{?}{=} 9$$

$$\sqrt{36} + 3 \stackrel{?}{=} 9$$

$$6 + 3 = 9 \checkmark$$

check: $x = -1$

$$\sqrt{2(-1)+18} + 3 \stackrel{?}{=} -1$$

$$\sqrt{16} + 3 \stackrel{?}{=} -1$$

$$4 + 3 \neq -1$$

Solution Set: $\{9\}$

Example 12. Algebraically solve the following equations. Clearly state each solution set.

(a) $\dfrac{8}{x^2-4} = \dfrac{x}{x+2}$

$$\frac{8}{x^2-4} = \frac{x}{x+2}$$

$$\frac{8}{(x-2)(x+2)} = \frac{x}{x+2}$$

$$\cancel{(x-2)}\cancel{(x+2)}\left(\frac{8}{\cancel{(x-2)}\cancel{(x+2)}}\right) = \left(\frac{x}{\cancel{x+2}}\right)(x-2)\cancel{(x+2)}$$

$$8 = x(x-2)$$

$$8 = x^2-2x$$

$$0 = x^2-2x-8$$

$$0 = (x-4)(x+2)$$

$x-4=0$ or $x+2=0$

$x=4$ $\quad$ $x=-2$

Check: $x=4$

$$\frac{8}{4^2-4} \stackrel{?}{=} \frac{4}{4+2}$$

$$\frac{8}{16-4} \stackrel{?}{=} \frac{4}{6}$$

$$\frac{8}{12} \stackrel{?}{=} \frac{4}{6}$$

$$\frac{2}{3} = \frac{2}{3} \checkmark$$

Check: $x=-2$

$$\frac{8}{(-2)^2-4} \stackrel{?}{=} \frac{-2}{-2+4}$$

$\dfrac{8}{4-4}$ is undefined

Solution set: $\{4\}$

(b) $5x^2-4x=-1$ (State all real and complex solutions)

$$5x^2-4x=-1$$

$$5x^2-4x+1=0$$

$$x = \frac{4 \pm \sqrt{(-4)^2-4(5)(1)}}{2(5)}$$

$$x = \frac{4 \pm \sqrt{16-20}}{10}$$

$$x = \frac{4 \pm \sqrt{-4}}{10}$$

$$x = \frac{4 \pm \sqrt{-1}\sqrt{4}}{10}$$

$$x = \frac{4 \pm 2i}{10}$$

$$x = \frac{2(2 \pm i)}{10}$$

$$x = \frac{2 \pm i}{5}$$

(In standard form, $x = \frac{2}{5} \pm \frac{i}{5}$)

Solution set: $\left\{ \frac{2+i}{5}, \frac{2-i}{5} \right\}$

or in standard form,

$\left\{ \frac{2}{5} + \frac{i}{5}, \frac{2}{5} - \frac{i}{5} \right\}$

8. Rational Exponents and Radical Notation

Example 13. Simplify each expression below using the rules of exponents. Write all simplified answers using positive, rational exponents. Assume all variables are positive.

(a) $\left(x^6\right)^{-1/2}$

$$\left(x^6\right)^{-1/2} = x^{6\cdot(-1/2)} = x^{-3} = \frac{1}{x^3}$$

(b) $\frac{x^{1/2}}{x^{1/8}}$

$$\frac{x^{1/2}}{x^{1/8}} = x^{1/2 - 1/8} = x^{4/8 - 1/8} = x^{3/8}$$

(c) $5x^{1/2} \cdot x^{2/3}$

$$5x^{1/2} \cdot x^{2/3} = 5x^{1/2 + 2/3} = 5x^{3/6 + 4/6} = 5x^{7/6}$$

Example 14. Write each expression below using radicals.

(a) $(2x)^{3/7}$

$$(2x)^{3/7} = \sqrt[7]{(2x)^3} = \sqrt[7]{8x^3}$$

(b) $2x^{3/7}$

$$2x^{3/7} = 2\sqrt[7]{x^3}$$

(c) $x^{-4/5}$

$$x^{-4/5} = \frac{1}{x^{4/5}} = \frac{1}{\sqrt[5]{x^4}}$$

Example 15. Simplify each expression below as much as possible. (All expressions are real numbers!)

(a) $4^{1/2}$

$$4^{1/2} = \sqrt{4} = 2$$

(b) $4^{-1/2}$

$$4^{-1/2} = \frac{1}{4^{1/2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

(c) $(-8)^{2/3}$

$$(-8)^{2/3} = \sqrt[3]{(-8)^2} = \sqrt[3]{64} = 4$$

Alternately:

$$(-8)^{2/3} = \left(\sqrt[3]{-8}\right)^2 = (-2)^2 = 4$$

9. Simplifying Rational Expressions

There are three important facts about rational expressions and equations:

- Only **factors** cancel.
- A rational expression is equivalent to another rational expression when BOTH the **numerator and denominator** are multiplied by the SAME expression.
- An equation with rational expressions is equivalent to another equation when BOTH **sides of the equation** are multiplied by the SAME expression.

Example 16. Simplify the complex fraction below as much as possible.

(a) $\dfrac{\frac{3}{x-3}}{\frac{x}{x-3}+2}$

$$\frac{\frac{3}{x-3}}{\frac{x}{x-3}+2} = \frac{\frac{3}{x-3}}{\frac{x}{x-3}+2} \cdot \frac{x-3}{x-3}$$

$$= \frac{\frac{3}{x-3}(x-3)}{\left(\frac{x}{x-3}+2\right)(x-3)}$$

$$= \frac{\frac{3}{\cancel{x-3}}\cancel{(x-3)}}{\frac{x}{\cancel{x-3}}\cancel{(x-3)}+2(x-3)}$$

$$= \frac{3}{x+2(x-3)}$$

$$= \frac{3}{x+2x-6}$$

$$= \frac{3}{3x-6}$$

$$= \frac{3}{3(x-2)}$$

$$= \frac{1}{x-2}$$

Example 17. Perform the indicated operation and simplify the expression as much as possible.

(a) $\dfrac{2x+4}{x^2-5x+6} \div \dfrac{x^2+4x+4}{4x-8}$

$$\frac{2x+4}{x^2-5x+6} \div \frac{x^2+4x+4}{4x-8} = \frac{2x+4}{x^2-5x+6} \cdot \frac{4x-8}{x^2+4x+4}$$

$$= \frac{2\cancel{(x+2)}}{\cancel{(x-2)}(x-3)} \cdot \frac{4\cancel{(x-2)}}{\cancel{(x+2)}(x+2)}$$

$$= \frac{8}{(x-3)(x+2)}$$

(b) $\dfrac{x+2}{x-3} - \dfrac{x+1}{x+4}$

LCD: $(x-3)(x+4)$

$$\frac{x+2}{x-3} - \frac{x+1}{x+4} = \frac{x+2}{x-3} \cdot \frac{x+4}{x+4} - \frac{x+1}{x+4} \cdot \frac{x-3}{x-3}$$

$$= \frac{(x+2)(x+4)}{(x-3)(x+4)} - \frac{(x+1)(x-3)}{(x+4)(x-3)}$$

$$= \frac{(x+2)(x+4) - (x+1)(x-3)}{(x-3)(x+4)}$$

$$= \frac{x^2+6x+8 - (x^2-2x-3)}{(x-3)(x+4)}$$

$$= \frac{x^2+6x+8 - x^2+2x+3}{(x-3)(x+4)}$$

$$= \frac{8x+11}{(x-3)(x+4)}$$