# Math 95 Final Exam Review

## 1. Evaluating Functions and Determining their Domain and Range

The **domain** of a function is the set of all possible inputs, which are typically $x$-values. The **range** of a function is the set of all possible outputs, which are typically $y$-values.

When determining the domain of a function based on the formula:

- Exclude any numbers that cause **division by zero**.
- Exclude any numbers that cause **the square root of a negative number** to occur.

**Example 1.** Algebraically determine the domain of the following functions. State each domain using set-builder notation AND interval notation. Then find $f(0)$ and $f(-2)$ for each function.

| Function | Domain (SB Notation) | Domain (Interval Notation) | $f(0)$ | $f(-2)$ |
|---|---|---|---|---|
| $f(x) = -5x + 7$ | | | | |
| $f(x) = \sqrt{-2x+3}$ | | | | |
| $f(x) = -3(x-1)^2 - 4$ | | | | |
| $f(x) = \frac{4x+1}{3x-6}$ | | | | |

**Example 2.** State the domain and range of each function below using interval notation.

Figure 1

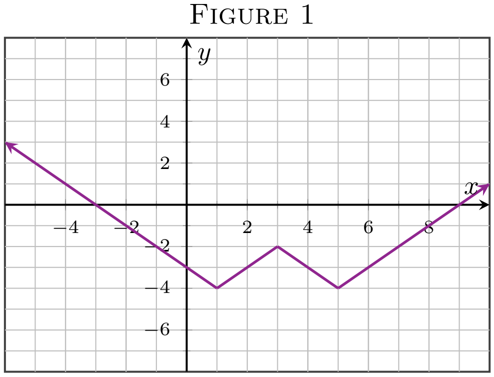
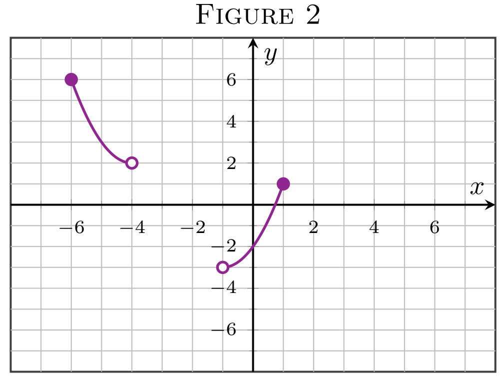



Figure 2




Domain:

Range:

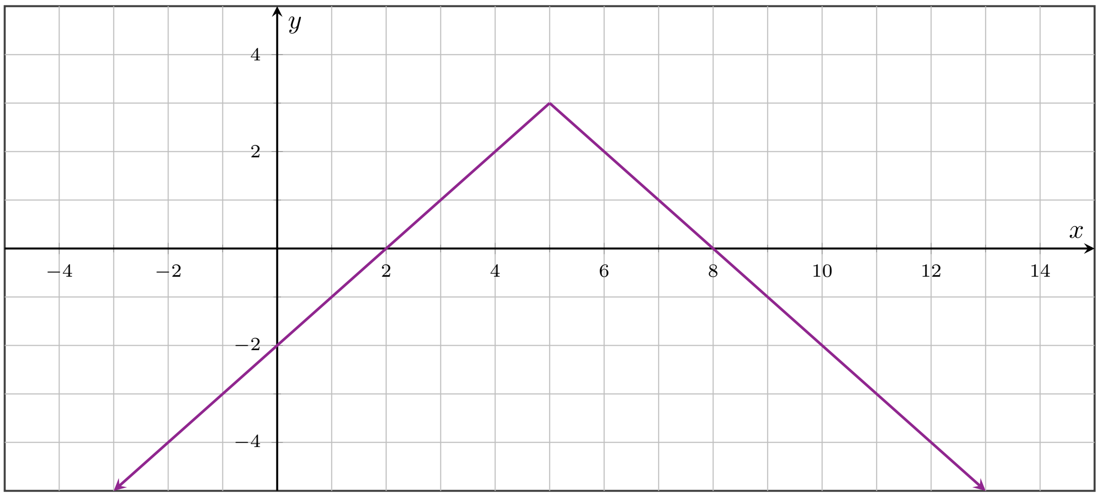
Domain:

Range:

## 2. Evaluating, Solving, and Determining Domain and Range Using the Graph of a Function

**Example 3.** Use the graph of $y = h(x)$ in Figure 3 to answer the following. State all solutions to equations in a solution set.

Figure 3




(a) State the domain of $h$.

(b) State the range of $h$.

(c) Evaluate $h(0)$.

(d) Solve $h(x) = 0$.

(e) State the following:

$x$-intercept(s):

$y$-intercept:

(f) Evaluate $h(1)$.

(g) Solve $h(x) = 1$.

(h) Solve $h(x) = 3$.

(i) Solve $h(x) \leq 2$.

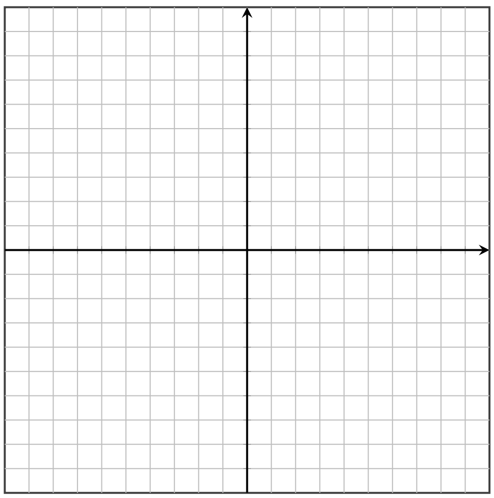
(j) Solve $h(x) > -3$.

## 3. Sketching Graphs of Functions By Hand

**Example 4.** Sketch the graph of $y = f(x)$ for each of the following functions **WITHOUT** a calculator. Clearly label any $x$-intercepts and $y$-intercepts.

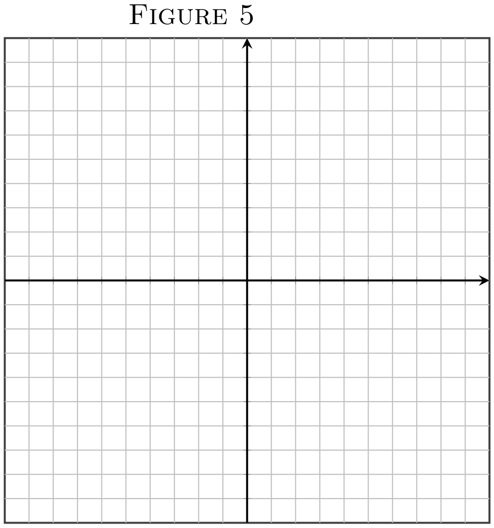
(a) $f(x) = 3x - 1$

Figure 4



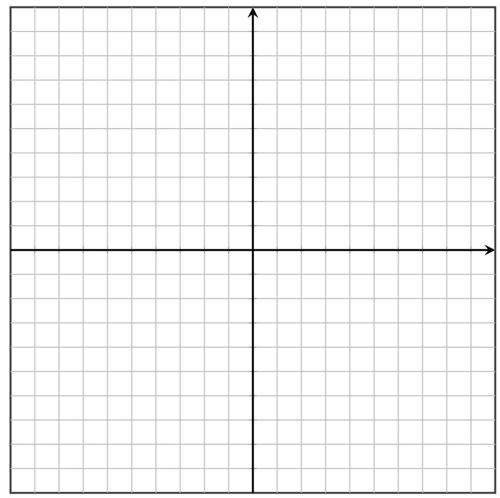
(b) $f(x) = -(x-1)^2 + 4$

Figure 5



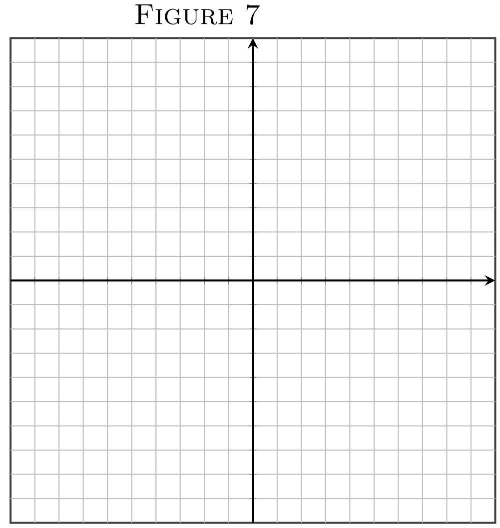
(c) $f(x) = (x+3)^2 - 1$

Figure 6



(d) $f(x) = 4$

Figure 7



## 4. Finding the Formula of a Linear Function

**Example 5.** Suppose that $y = g(x)$ is a linear function and that $g(-2) = 11$ and $g(5) = -3$. Find the algebraic formula for $g(x)$.

**Example 6.** Suppose that $y = h(t)$ is a linear function and that $h(4) = 6$ and $h(10) = 9$. Find the algebraic formula for $h(t)$.

## 5. Completing the Square

**Example 7.** Write the quadratic equation $y = x^2 + 12x + 5$ in vertex form by completing the square. Identify the vertex.

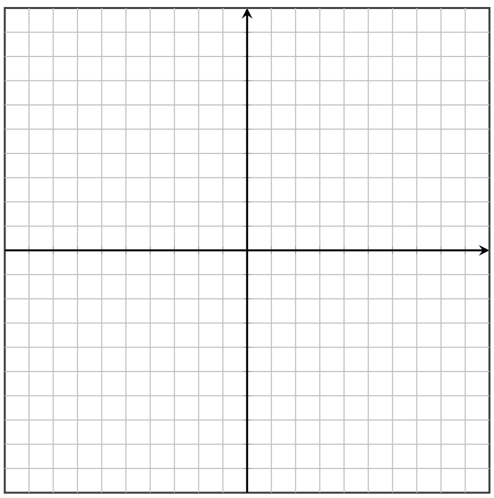
**Example 8.** Solve quadratic equation $x^2 - 6x + 1 = -10$ using completing the square. Clearly state all real and complex solutions in a solution set.

## 6. Graphing Functions and Solving Equations with your Graphing Calculator

**Example 9.** Sketch the graph of $y = f(x)$ for each of the following functions **WITH** the help of your calculator. Clearly label any $x$-intercepts and $y$-intercepts.

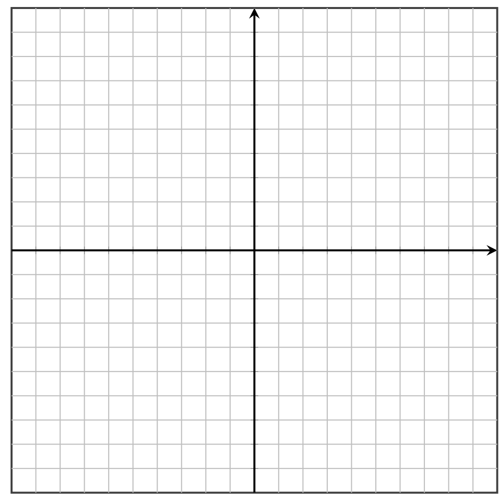
(a) $f(x) = \sqrt{x+4} - 3$

Figure 8



(b) $f(x) = \frac{-8}{x+4}$

Figure 9



**Example 10.** **Graphically** solve the following equations and inequalities using your graphing calculator. Clearly state each solution set. Round all solutions accurate to three decimal places.

(a) $-16x^2 + 64x + 3 = 10$

(b) $|-2x + 7| > 1$

(c) $-16x^2 + 64x + 3 = 0$ (Use the ZERO feature)

(d) $x^{5/3} = x^{1/2} + 1$

## 7. Solving Equations Algebraically

**Example 11.** **Algebraically** solve the following equations and inequalities. Clearly state each solution set.

(a) $-8 < -4x + 12 \leq 16$

(b) $\sqrt{2x + 18} + 3 = x$

**Example 12.** **Algebraically** solve the following equations. Clearly state each solution set.

(a) $\dfrac{8}{x^2 - 4} = \dfrac{x}{x+2}$

(b) $5x^2 - 4x = -1$ (State all real and complex solutions)

## 8. Rational Exponents and Radical Notation

**Example 13.** Simplify each expression below using the rules of exponents. Write all simplified answers using positive, rational exponents. Assume all variables are positive.

(a) $\left(x^6\right)^{-1/2}$ (b) $\frac{x^{1/2}}{x^{1/8}}$ (c) $5x^{1/2} \cdot x^{2/3}$

**Example 14.** Write each expression below using radicals.

(a) $(2x)^{3/7}$ (b) $2x^{3/7}$ (c) $x^{-4/5}$

**Example 15.** Simplify each expression below as much as possible. (All expressions are real numbers!)

(a) $4^{1/2}$ (b) $4^{-1/2}$ (c) $(-8)^{2/3}$

## 9. Simplifying Rational Expressions

There are three important facts about rational expressions and equations:

- Only **factors** cancel.
- A rational expression is equivalent to another rational expression when BOTH the **numerator and denominator** are multiplied by the SAME expression.
- An equation with rational expressions is equivalent to another equation when BOTH **sides of the equation** are multiplied by the SAME expression.

**Example 16.** Simplify the complex fraction below as much as possible.

(a) $\dfrac{\dfrac{3}{x-3}}{\dfrac{x}{x-3}+2}$

**Example 17.** Perform the indicated operation and simplify the expression as much as possible.

(a) $\dfrac{2x+4}{x^2-5x+6} \div \dfrac{x^2+4x+4}{4x-8}$

(b) $\dfrac{x+2}{x-3} - \dfrac{x+1}{x+4}$