Name: _____ 100 POINTS

Practice Math 65 Final Exam

Part I: No Calculator

Show all your work so that:

- someone who wanted to know *how* you found your answer can clearly see how.
- if you make a mistake, I can see where it happened and determine how much partial credit you should be awarded.

You may use scratch paper, but all necessary work must be written on this exam. Simplify all fractions as much as possible. The entire exam is closed-note, closed-book. You may not use your calculator or any other electronic device on this part of the exam. Take your time, because you have plenty to spare. Check your answers ☺.

1. Solve the following systems of linear equations by graphing. State your solution and your solution set for each problem. Check each answer.

a) $\begin{cases} y &= \frac{1}{2}x \\ y &= 2x - 3 \end{cases}$

b) $\begin{cases} 2x - 3y &= 12 \\ 2x + y &= 4 \end{cases}$

2. Solve the system of equations using the addition method. State your solution and your solution set.

$$\begin{cases} -4x + 8y &= 0 \\ 7x + 2y &= -8 \end{cases}$$

3. Solve the system of equations using the substitution method. State your solution and your solution set.

$$\begin{cases} 4x + y &= 5 \\ 12x + 3y &= 15 \end{cases}$$

4. Let $f(x) = x^2 - 4x + 3$. Calculate the following.

a) $f(2)$ b) $f(0)$ c) $f(-3)$

5. Determine if the following relations are functions. Then state the domain and range of each relation.

a) $\{(1,3),(2,5),(4,5)\}$ b) $\{(1,1),(1,4),(3,9)\}$

6. Using appropriate function notation, complete the following for the function h in Figure 1.

a) Find $h(0)$.

b) What is the function value at -4?

c) What is the solution to the equation $h(x) = -1$?

d) Find the equation of the line in Figure 1 and write the equation in slope-intercept form.

e) Write the formula for $h(x)$.

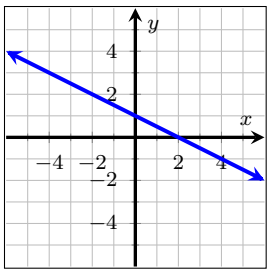


FIGURE 1. $y = h(x)$

7. Add or subtract the following polynomials.

a) $(6x^3 + 5x^2 - 7x) - (9x^3 + x^2 - 6x)$

b) $(5y^3 - 9y^2 - 4) + (3y^3 - 12y^2 - 5)$

c) $(5x^2y^3 - xy + 4y^2) + (8x^2y^3 - 6xy - 2y^2)$

8. Multiply/expand and fully simplify each polynomial.

a) $(3x + 7)(2x - 5)$

b) $2x^3(4x^5 - 5x^2)$

c) $(2x - 3)^2$

d) $(4x - 3)(4x + 3)$

e) $(3x + 2)(6x^2 - 4x + 2)$

9. Write the following in scientific notation.

a) $250,000$

b) $(5 \times 10^{-2})(6 \times 10^{5})$

10. Simplify the following expressions.

a) x^2x^5

b) $(-3x^5)^{-2}$

c) $\dfrac{y^2}{y^{10}}$

d) $\dfrac{6x^5 + 4x^3 + 10x^2}{2x^2}$

e) $(x^2)^5$

f) $(-2x^2)^0$

g) $\left(\dfrac{2}{x}\right)^{-1}$

h) $\left(\dfrac{8x^5}{4x^3}\right)^3$

11. Factor the following expressions completely. If the expression is prime, give appropriate justification. You are encouraged (but not required) to check your answers.

a) $x^2 + 9x + 20$

b) $3x^2 - 7x - 6$

c) $4x^2 + 4x + 4$

d) $x^2 - 5x - 5$

e) $3x^2 - 9x + 6$

f) $9x^2 - 6x + 1$

12. Factor the following expressions completely. If the expression is prime, give appropriate justification. You are encouraged (but not required) to check your answers.

a) $18xy^2 - 8x$

b) $2x^3 - 18x$

c) $64x^2 - 25$

d) $2x^3 + 18x$

13. Simplify the following radical expressions as much as possible.

a) $\sqrt{32}$ b) $\sqrt{50}+\sqrt{75}$ c) $\sqrt{300}$ d) $\sqrt{48}+\sqrt{75}$

14. Rationalize the denominator.

a) $\dfrac{2}{\sqrt{8}}$ b) $\dfrac{6}{\sqrt{5}}$

15. Solve the following quadratic equations by factoring. State the solution set and check your answer.

a) $x^2-2x-15=0$ b) $y(y+8)=16(y-1)$

16. Solve the following quadratic equations using the square root property. State the solution set.

a) $4y^2=49$ b) $(x-5)^2=9$

17. Solve the following quadratic equations using the quadratic formula. State the solution set.

a) $2x^2+6x-4=0$ b) $3x^2+x=4$

18. Solve the following quadratic equations using the method of your choice. State the solution set.

a) $5x^2-5=0$ b) $x(3x+8)=-5$

c) $(x+5)(x-1)=0$ d) $4x^2+8x=-4$

19. Graph the parabola whose equation is $y=-x^2+4$. Find the x-intercept(s), the y-intercept, the vertex, and at least one extra point.

20. Graph the parabola whose equation is $y=x^2-2x-3$. Find the x-intercept(s), the y-intercept, the vertex, and at least one extra point.

Name: _____

Practice Math 65 Final Exam

Part II: Calculator Permitted

You may use a calculator (basic, scientific, or graphing), but may not use any other electronic device. Show all your work so that:

- someone who wanted to know *how* you found your answer can clearly see how.
- if you make a mistake, I can see where it happened and determine how much partial credit you should be awarded.

The calculator should only be used at the end of your problem-solving process, to calculate some decimal value. Where appropriate, round to four significant digits.

21. A charity sells tickets for a fundraising dinner. Each adult's ticket costs 10 dollars and each child's ticket costs 5 dollars. A total of \$1050 was raised by selling 130 tickets. How many adult and child tickets were sold?

22. Nutritional information is given for one ounce of cheddar cheese and one apple in Table 1. How many of each should be eaten to get exactly 305 calories and 12 grams of fat?

Table 1

	1 oz Cheddar	1 Apple
Fat (g)	9	0.5
Calories	100	120

23. Round the following irrational numbers to the nearest thousandth.

a) $\sqrt{97}$ b) $\dfrac{\sqrt{3}}{5}$ c) $\dfrac{1-\sqrt{5}}{7}$

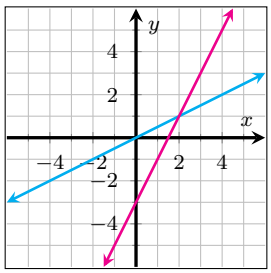
24. The distance, d, in feet, that an object falls in t seconds is modeled by the formula $d = 16t^2$. If you drop a rock from a cliff that is 448 feet above the water, how long will it take for the rock to hit the water? Give an **exact, simplified** answer. Then round your answer to the nearest tenth.

25. A garden bed is 10 feet long and 4 feet wide. What is the diagonal length? Give an **exact, simplified** answer. Then round your answer to the nearest tenth. *Hint: Use the Pythagorean Theorem, which states that for a right triangle with side lengths a, b, and c the following holds:* $a^2 + b^2 = c^2$.

Answers

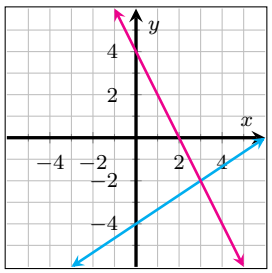
Note: Only the short answers to each problem are listed here. All work and justification should be shown on the exam (unless the problem specifically states otherwise).

1. a)



Solution Set: $\{(2,1)\}$

b)



Solution Set: $\{(3,-2)\}$

2. Solution Set: $\left\{\left(-\frac{1}{2},-1\right)\right\}$

3. You should obtain an identity. Thus all ordered pairs on each line are solutions. Solution Set: $\{(x,y)\,|4x+y=5\}$

4. a) -1 b) 3 c) 24

5. a) Function
Domain: $\{1,2,4\}$
Range: $\{3,5\}$

b) Not a function
Domain: $\{1,3\}$
Range: $\{1,4,9\}$

6. a) $h(0)=1$ b) 3 c) $x=4$
d) $y=-\frac{1}{2}x+1$ e) $h(x)=-\frac{1}{2}x+1$

7. a) $-3x^4+4x^2-x$ b) $8y^3-21y^2-9$ c) $13x^2y^3-7xy+2y^2$

8. a) $6x^2-x-35$ b) $4x^2-12x+9$ c) $8x^8-10x^5$
d) $16x^2-9$ e) $18x^3-2x+4$

9. a) 2.5×10^5 b) 3×10^4

10. a) x^7 b) $\frac{1}{9x^{10}}$ c) $\frac{1}{y^8}$ d) $3x^3+2x+5$
e) x^{10} f) 1 g) $\frac{x}{2}$ h) $8x^6$

11. a) $(x+4)(x+5)$ b) $(x-3)(3x+2)$ c) $4(x^2+x+1)$
d) prime e) $3(x-1)(x-2)$ f) $(3x-1)^2$

12. a) $2x(3y-2)(3y+2)$ b) $2x(x-3)(x+3)$ c) $(8x-5)(8x+5)$
d) $2x(x^2+9)$

13. a) $4\sqrt{2}$ b) $5\sqrt{2}+5\sqrt{3}$ c) $10\sqrt{3}$ d) $9\sqrt{3}$

14. a) $\dfrac{\sqrt{2}}{2}$ b) $\dfrac{6\sqrt{5}}{5}$

15. a) $\{5,-3\}$ b) $\{4\}$

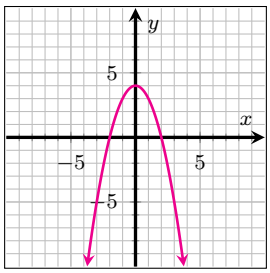
16. a) $\left\{\frac{7}{2},-\frac{7}{2}\right\}$ b) $\{2,8\}$

17. a) $\left\{\frac{-3+\sqrt{17}}{2},\frac{-3-\sqrt{17}}{2}\right\}$ b) $\left\{-\frac{4}{3},1\right\}$

If not simplified: $\left\{\frac{-6+\sqrt{68}}{4},\frac{-6-\sqrt{68}}{4}\right\}$

18. a) $\{1,-1\}$ b) $\left\{-1,-\frac{5}{3}\right\}$ c) $\{-5,1\}$ d) $\{-1\}$

19.



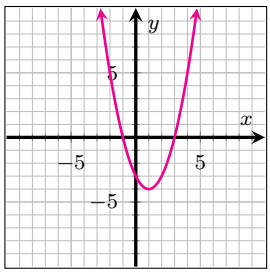
x-intercepts: $(-2,0)$ and $(2,0)$

y-intercept: $(0,4)$

vertex: $(0,4)$

extra point $(3,-5)$

20.



x-intercepts: $(3,0)$ and $(-1,0)$

y-intercept: $(0,-3)$

vertex: $(1,-4)$

extra point $(4,5)$

21. Let x be the number of adult tickets sold and let y be the number of child tickets sold.

System: $\begin{cases} x+y=130 \\ 10x+5y=1050 \end{cases}$

Solution: $(80,50)$

They sold 80 adult tickets and 50 child tickets.

22. Let c be the number of ounces of cheddar and let a be the number of apples eaten.

System: $\begin{cases} 9c+0.5a=12 \\ 100c+120a=305 \end{cases}$

Solution: $c=1.25, a=1.5$

You need to eat 1.25 ounces of cheddar and 1.5 apples to consume 12 grams of fat and 305 calories.

23. a) $\sqrt{97}\approx 9.849$ b) $\frac{\sqrt{3}}{5}\approx 0.346$ c) $\frac{1-\sqrt{5}}{7}\approx -0.177$

24. $t=\pm\sqrt{28}$ (exact), $t=\pm 2\sqrt{7}$ (exact and simplified), $t\approx\pm 5.3$

It takes approximately 5.3 seconds for the rock to hit the water.

25. $d=\pm\sqrt{116}$ (exact), $d=\pm 2\sqrt{27}$ (exact and simplified), $d\approx\pm 10.8$

The length of the diagonal is about 10.8 feet.