Chapter 3 – Fractions

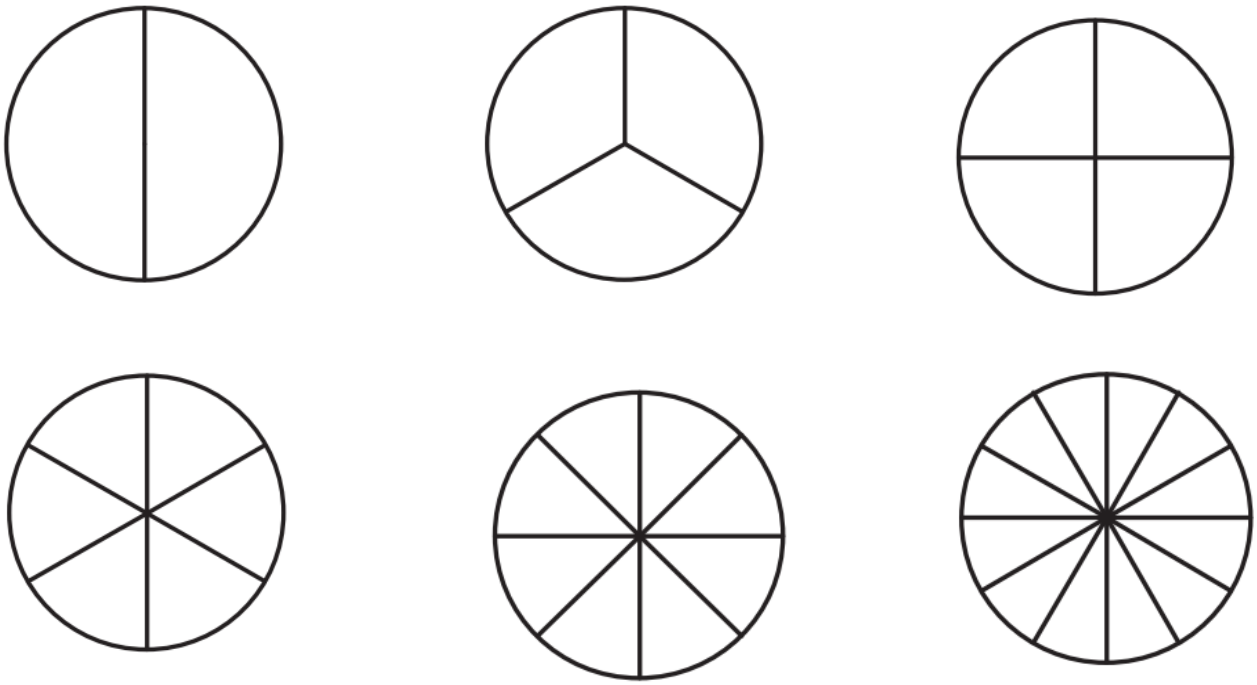
Contents

Chapter 3 – Fractions 1
Introduction to Fractions with Manipulatives 2
Introduction to Fractions - Practice 5
Divisibility Rules 8
Prime and Composite Numbers 9
Multiplying Fractions 10
Multiplying and Cross-Canceling Fractions 12
Dividing Fractions 14
Adding and Subtracting Fractions Activity 17
Finding the Least Common Denominator (LCD) 21
Practice Adding and Subtracting Fractions 22
Fraction Puzzles 23
Mixed Numbers 24
Mixed Operations with Fractions 26
Order of Operations with Fractions 28

1. Take all of the fraction pieces out of the bag, sort them by color and put them into circles. There should be enough pieces of each color to make one whole.

$$\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}} = \frac{\text{Number of pieces}}{\text{Total pieces}}$$

Write the fraction for each piece shown. The numerator is equal to one and the denominator represents the number of pieces in the whole.



2. How many fourths make a half? Find the fraction pieces and draw a picture to show this.

3. How many eighths make a fourth? Find the fraction pieces and draw a picture to show this.

4. How many eighths make a half? Find the fraction pieces and draw a picture to show this.

5. How many twelfths make a third? Find the fraction pieces and draw a picture to show this.

6. Is any piece equivalent to a third plus a sixth? If so, which one? Find the fraction pieces and draw a picture to show this.

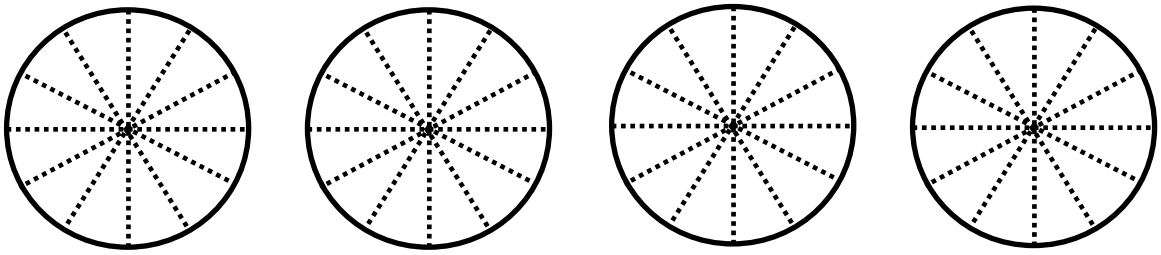
7. Can you add 2 different pieces to make two-thirds? If so, which ones? Find the fraction pieces and draw a picture to show this.

8. Can you find 3 different ways to make three-quarters? Find the fraction pieces and draw a picture to show this.

9. How many different ways can you make one-half? Find the fraction pieces and draw a picture with a label for each equivalent fraction.

10. Find the fraction pieces listed below and put them in order from smallest to largest. In the circles below, shade each fraction and write the fraction underneath.

$\frac{1}{3}, \frac{1}{6}, \frac{1}{4}, \frac{1}{2}$



11. In between which two circles would fifths be placed? In between which two circles would sevenths be placed? Draw arrows and label where the fifths and sevenths would be placed.

12. Give a rule for ordering fractions of this kind (with 1 in the numerator).

Introduction to Fractions - Practice **Name** _____

Divide the object into the given fractions. Then shade the fraction listed.

1. Halves, $\frac{1}{2}$



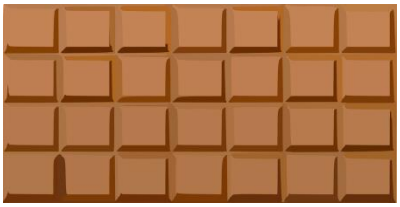
2. Eighths, $\frac{3}{8}$



3. Fourths, $\frac{3}{4}$

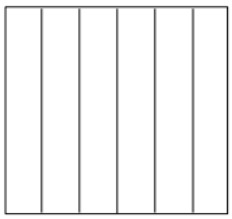


4. Sevenths, $\frac{5}{7}$

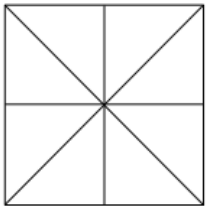


Shade the figure so that it represents the given fraction. Then write the corresponding equivalent fraction.

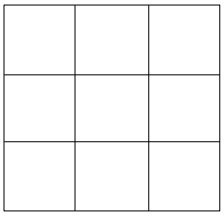
5. $\frac{1}{2} =$



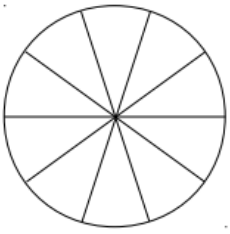
6. $\frac{3}{4} =$



7. $\frac{2}{3} =$



8. $\frac{4}{5} =$



"Pizza Anyone" is licensed under CC BY-NC 4.0 / A derivative from the original work.

Build each fraction to make an equivalent fraction with the given denominator. Multiply the numerator and the denominator by the same number.

9. $\frac{1}{2} \cdot \frac{\square}{\square} = \frac{\square}{10}$

10. $\frac{3}{4} \cdot \frac{\square}{\square} = \frac{\square}{12}$

11. $\frac{2}{5} \cdot \frac{\square}{\square} = \frac{\square}{20}$

12. $\frac{6}{7} \cdot \frac{\square}{\square} = \frac{\square}{42}$

13. $\frac{3}{2} \cdot \frac{\square}{\square} = \frac{\square}{18}$

14. $-\frac{5}{12} \cdot \frac{\square}{\square} = -\frac{\square}{36}$

Reduce each fraction to lowest terms. Divide a common factor out of the numerator and denominator.

15. $\frac{2}{4}$

16. $\frac{15}{30}$

17. $\frac{30}{48}$

18. $\frac{15}{45}$

19. $\frac{14}{36}$

20. $-\frac{16}{48}$

Simplify each fraction or state that it cannot be simplified.

21. $\frac{8}{15}$

22. $-\frac{2}{42}$

23. $\frac{16}{36}$

24. $\frac{48}{82}$

25. $\frac{49}{7}$

26. $-\frac{96}{102}$

When a fraction is negative we can write the negative sign in front of the fraction, on the numerator or ono the denominator. Write each fraction in two other ways.

27. $-\frac{3}{4}$

28. $\frac{-6}{5}$

29. $\frac{2}{-3}$

It is easier to compare fractions when the denominators are the same. Build one of the fractions so they have the same denominator. Then write >, < or = to make a true statement.

30. $\frac{2}{5} \square \frac{3}{10}$

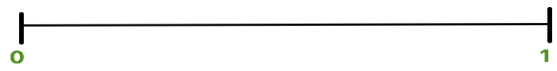
31. $\frac{1}{3} \square \frac{5}{12}$

32. $\frac{15}{18} \square \frac{5}{6}$

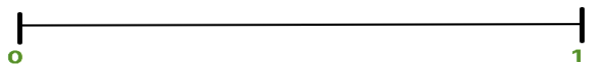
33. Build each fraction to a common denominator of 45, then write from smallest to largest: $\frac{4}{9}, \frac{3}{5}, \frac{2}{15}, \frac{1}{3}$

Draw a line and label where the fraction belongs on the number line. (Hint: Divide the number line into equal sections)

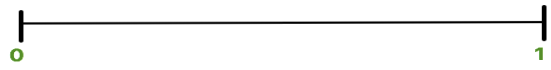
34. $\frac{1}{3}$



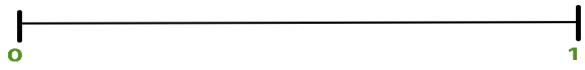
35. $\frac{3}{4}$



36. $\frac{5}{6}$



37. $\frac{1}{8}$



Divisibility Rules

Name _____

For Simplifying Fractions

A number is divisible by

2 if it is even

3 if the sum of its digits is divisible by 3

5 if its last digit is 0 or 5

9 if the sum of its digits is divisible by 9

10 if its last digit is 0

1. Is 930 divisible by

2?

3?

5?

9?

10?

2. Is -783 divisible by

2?

3?

5?

9?

10?

3. Is 43,905 divisible by

2?

3?

5?

9?

10?

4. Is 16,312 divisible by

2?

3?

5?

9?

10?

5. How are the divisibility rules useful for working with fractions?

Prime and Composite Numbers

Name _____

For Simplifying Fractions

1. A prime number has exactly two factors. It is only divisible by one and itself. List some examples of prime numbers.

2. A composite number has more than two factors. List some examples of composite numbers.

3. There is one number that is neither prime nor composite. What is it?

4. Cross out all of the composite numbers and circle all of the prime numbers in the table.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

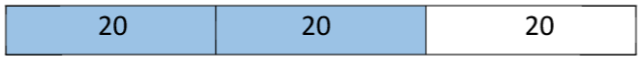
5. What patterns can you observe in the table?

6. How is the concept of prime and composite useful for reducing fractions?

 Name _____

Multiplying a fraction by a whole number

1. The following picture can be used to show that two-thirds of 60 is 40:



Write this result as a mathematical equation:

2. Draw a similar picture as above to find three-fourths of 12.

Write the result as a mathematical equation:

For each problem, draw a picture and write a multiplication problem involving fractions. Then find the answer and state it in a complete sentence.

3. In 1996, the state of Oregon passed a referendum to require a "supermajority" of three-fifths of the votes in the legislature to pass a tax increase bill. If all 90 of Oregon's state legislators vote on a proposed tax increase, how many must approve for the bill to pass?

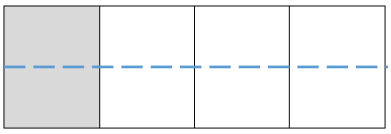
4. A recipe calls for one-half of a tablespoon of olive oil per serving. How much olive oil is required to make 5 servings?

5. Currently there are 27 amendments to the U.S. Constitution. In order to pass a constitutional amendment, it must be approved by three-fourths of the state legislatures. How many state legislatures must approve?

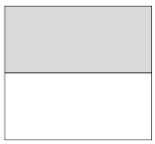
Use the diagram to find the answer and then write the corresponding multiplication statement.

6. What is $\frac{1}{2}$ of $\frac{1}{4}$?

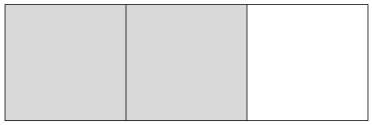
$\frac{1}{2} \cdot \frac{1}{4} =$



7. What is $\frac{2}{3}$ of $\frac{1}{2}$?



8. What is $\frac{3}{4}$ of $\frac{2}{3}$?



For each problem, draw a picture and write a multiplication problem involving fractions. Then find the answer and state it in a complete sentence.

9. A recipe to make 3 dozen cookies requires one-fourth of a cup of butter. How much butter should you use if you only want to make a dozen cookies?

10. Three members of the PCC math department purchased a lottery ticket and won the grand prize. If state and federal taxes combine to get two-fifths of the money, and they are going to split the remaining money equally, what fraction of the grand prize will each member receive?

11. A survey of Portlanders found that seven-tenths of them own pets, and that two-thirds of all pet owners have dogs. What fraction of Portlanders own dogs?

Multiplying and Cross-Canceling Fractions

Name _____

Multiply First

Multiply the fractions together first and then simplify the result.

1. $\frac{3}{5}\cdot\frac{7}{6}$

2. $-\frac{11}{2}\cdot\frac{4}{3}$

3. $\left(-\frac{3}{8}\right)\cdot\left(-\frac{2}{9}\right)$

4. $-3\cdot\frac{1}{6}$

5. $\frac{1}{4}\cdot 28$

6. $\frac{2}{3}\cdot\frac{5}{4}\cdot\frac{9}{10}$

Cancel Common Factors First

Do the same problems by canceling common factors first and then multiplying.

7. $\frac{3}{5}\cdot\frac{7}{6}$

8. $-\frac{11}{2}\cdot\frac{4}{3}$

9. $\left(-\frac{3}{8}\right)\cdot\left(-\frac{2}{9}\right)$

10. $-3\cdot\frac{1}{6}$

11. $\frac{1}{4}\cdot 28$

12. $\frac{2}{3}\cdot\frac{5}{4}\cdot\frac{9}{10}$

Which method do you prefer and why?

13. $\frac{15}{4}\cdot\frac{1}{9}$

14. $\left(\frac{3}{7}\right)\left(\frac{14}{9}\right)$

15. $\frac{36}{45}\cdot\left(-\frac{5}{6}\right)$

16. $-\frac{7}{8}\cdot\frac{2}{21}$

17. $\frac{11}{6}\cdot\frac{5}{3}$

18. $\left(-\frac{4}{7}\right)\cdot\left(-\frac{3}{8}\right)$

19. $-4\cdot\frac{1}{16}$

20. $30\left(\frac{2}{5}\right)$

21. $\frac{3}{8}\cdot 24$

22. $\frac{1}{2}\cdot\frac{2}{5}\cdot\frac{5}{3}$

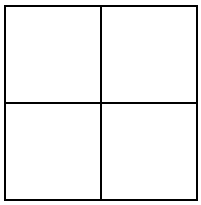
23. $\left(-\frac{18}{3}\right)\cdot\frac{4}{9}\cdot\left(-\frac{3}{8}\right)$

24. $\left(-\frac{3}{16}\right)\cdot\left(-\frac{9}{7}\right)\cdot\left(-\frac{4}{27}\right)$

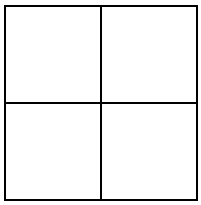
 Name _____

The relationship between multiplying and dividing fractions

1. Divide one fourth in two and shade that region. What fraction do you have?



2. Now shade half of a fourth. What fraction do you have?



3. Problem 1 is a division problem.

$\frac{1}{4} \div 2$ or $\frac{1}{4} \div \frac{2}{1} =$

4. Problem 2 is a multiplication problem.

$\frac{1}{4} \cdot \frac{1}{2} =$

5. Dividing by 3 is the same as multiplying by what fraction?

6. Dividing by 5 is the same as multiplying by what fraction?

7. What is the relationship between dividing and multiplying fractions?

Practice dividing by multiplying by the reciprocal.

8. $\frac{2}{3} \div \frac{1}{2}$

9. $\frac{1}{3} \div \left(-\frac{1}{2} \right)$

10. $-\frac{3}{4} \div \frac{3}{2}$

11. $\frac{11}{16} \div \left(-\frac{9}{16}\right)$

12. $-1 \div \frac{1}{4}$

13. $-\frac{1}{7} \div \left(-\frac{5}{6}\right)$

Mixed Practice

14. $\frac{7}{10} \cdot \frac{20}{21}$

15. $-9 \cdot \frac{1}{8}$

16. $-\frac{28}{15} \div \frac{21}{10}$

17. $-\frac{3}{4} \div 4$

For each problem, show your thinking in pictures, symbols and/or words. Show your steps and write your answer in a complete sentence.

18. A recipe calls for $\frac{3}{4}$ of a cup of flour and you are tripling the batch. How many cups of flour do you need?

19. A survey found that seven-tenths of Portlanders own pets, and that two-thirds of all pet owners have dogs. What fraction of Portlanders own dogs?

20. A recipe to make 3 dozen cookies requires one-fourth of a cup of butter. How much butter should you use if you only want to make a dozen cookies?

21. How many servings are there in an 8-pound roast if the suggested serving size is $\frac{2}{3}$ pound?

22. Three members of the PCC math department purchased a lottery ticket and won the grand prize. If state and federal taxes combine to get two-fifths of the money, and they are going to split the remaining money equally, what fraction of the grand prize will each member receive?

Adding and Subtracting Fractions Activity **Name** _____

With Fraction Circle Manipulatives

Like Denominators

1. Below are two apple pies. If you and 7 of your friends are going to eat these pies, you need 8 equal pieces. Draw lines on each pie so that each one has four equal parts.





2. Two of your friends each eat a piece from the left pie. Shade the two pieces of the pie that have been eaten.

3. Three of your friends eat pieces out of the right pie. Shade the three pieces of the pie that have been eaten.

4. You and the remaining friends decide you are not hungry and aren't going to eat the rest of the pie. They leave the remaining pie with you to take home to your kids.

a. What fraction of the left pie is still remaining? (Don't reduce the fraction yet.)

b. What fraction of the right pie is still remaining?

c. Find the fraction circle pieces to represent the amount of pie that is remaining and put them together. Draw a picture of what it would look like if you put all the remaining pieces into one pie pan.

d. Create an addition model that would represent adding the leftover pieces from the left and right pies together. (Do not use a reduced fraction for the left pie). How much pie is remaining?

$$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$$

Left Pie Right Pie Total Pie Remaining

e. What do you notice about the denominators of all the fractions? *(including the answer)*

f. What did you do with the numerators to get the final answer?

5. Now let's just look at only the pie on the right. You cut the pie into 4 equal parts. Before anyone ate any of the pie you had 4 parts. The fraction corresponding to the uneaten original pie is $\frac{4}{4}$.

a. Write a fraction for how much of the right pie was eaten by your friends.

b. Write a subtraction problem to model how much pie is left on the right pie.

$$\frac{\square}{\square} - \frac{\square}{\square} =$$

Whole Pie Right Pie Eaten Fraction Remaining

c. What do you notice about the denominators of all the fractions? (including the answer)

d. What did you do with the numerators to get the final answer?

6. Find the coordinating fraction circle pieces for each addition or subtraction problem below. Use them to compute your answer. Draw a picture to represent each problem and write the answer.

a. $\frac{1}{3}+\frac{1}{3}$ b. $\frac{4}{6}-\frac{1}{6}$ c. $\frac{2}{8}+\frac{3}{8}$

Unlike Denominators

7. Now let's look at some pizza! You bought the pepperoni mini-pizza on the left and cut it into three equal pieces. Find the fraction circle pieces to model the left pizza and draw lines on the picture to make three equal parts.

8. Your friend bought the veggie mini-pizza on the right and cut it into two equal pieces. Find the fraction circle pieces to model the right pizza and draw a line on the picture to make two equal parts.





9. You ate two pieces of the pepperoni pizza. Shade the two of the pieces on the picture and remove the fraction circle pieces. What fraction of the pizza is remaining?

10. Your friend ate one piece of the veggie pizza. Shade the piece that was eaten and remove the fraction circle piece. What fraction of the pizza is remaining?

11. Put the remaining fraction circle pieces together to represent the total amount of pizza that is left. Draw a picture of what it would look like if you put all of the remaining pieces together in one pie pan.

12. Is it easy to tell what fraction of the pizza is remaining? Why or why not? Discuss in your group what could be done to make it easier to see what fraction of the pizza is left.

13. The problem is that halves and thirds are not the same size so we can't add them together. If you haven't already, find a smaller size fraction piece that you can use to replace both the third and the half? What is the denominator?

14. Now can you tell what fraction of a pizza is leftover? If so, you just found a common denominator.

15. On the pizzas below, draw lines on each pizza to represent the smaller size pieces that you found and shade the parts that have been eaten.





16. Write the equivalent fraction of each pizza that is now cut into smaller pieces. Then add them together.

$$\frac{\square}{\square} + \frac{\square}{\square} = $$

Left Pizza Right Pizza Total Fraction Remaining

17. What does the denominator of a fraction represent? What is needed to add or subtract fractions? (Discuss this in your group then write it down.)

18. Think about your answer in problem 16. How can you make it so the fractions have what is needed in order to add or subtract them? (Discuss this in your group then write it down.)

"Pizza Anyone" is licensed under CC BY-NC 4.0 / A derivative from the original work.

Finding the Least Common Denominator (LCD) **Name** _____

- Make the denominators match fraction circles

Fill in the table with multiples of each denominator in order. Look for the smallest common multiple. The first one is done for you. Then build each fraction up to have the common denominator so you can add or subtract.

1. $\frac{1}{3}+\frac{2}{7}$

3	6	9	12	15	18	21
7	14	21				

Least Common Denominator (LCD) = 21

$$\frac{1}{3}\cdot\frac{\boxed{7}}{\boxed{7}}+\frac{2}{7}\cdot\frac{\boxed{3}}{\boxed{3}}$$

$$=\frac{\boxed{}}{\boxed{21}}+\frac{\boxed{}}{\boxed{21}}$$

$$=\frac{\boxed{}}{\boxed{21}}$$

2. $\frac{5}{9}+\frac{7}{18}$

9						
18						

LCD =

$$\frac{5}{9}\cdot\frac{\boxed{}}{\boxed{}}+\frac{7}{18}\cdot\frac{\boxed{}}{\boxed{}}$$

$$=\frac{\boxed{}}{\boxed{}}+\frac{\boxed{}}{\boxed{}}$$

$$=\frac{\boxed{}}{\boxed{}}$$

3. $\frac{1}{10}-\frac{1}{12}$

10						
12						

LCD =

Shortcuts: How do you find the LCD when

- the denominators have no factors in common?
- one denominator is a multiple of the other denominator?
- the denominators have a common factor?

Practice Adding and Subtracting Fractions **Name** _____

Perform the indicated operation(s).

1. $\frac{1}{2}+\frac{2}{3}$

2. $\frac{3}{8}-\frac{1}{3}$

3. $\frac{3}{8}-\left(-\frac{1}{2}\right)$

4. $\frac{7}{12}-\frac{4}{15}$

5. $-2+\frac{5}{7}$

6. $-\frac{5}{8}-\left(-\frac{7}{6}\right)$

7. $\frac{18}{11}-\frac{7}{11}$

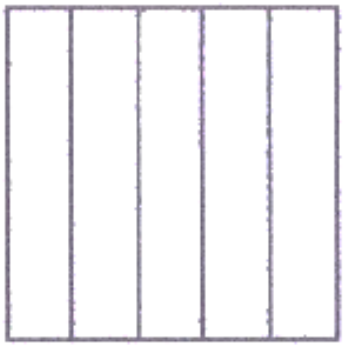
8. $\left(-\frac{1}{2}\right)+\frac{8}{16}$

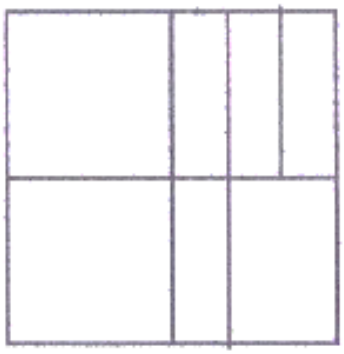
9. $\frac{3}{4}+\frac{1}{6}-\frac{7}{3}$

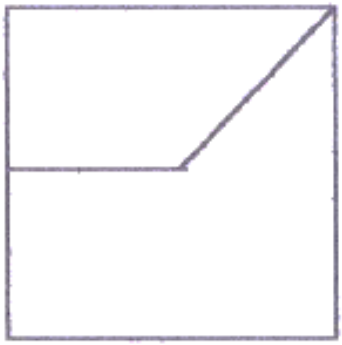
10. $\frac{4}{5}-\frac{2}{3}+\frac{7}{15}$

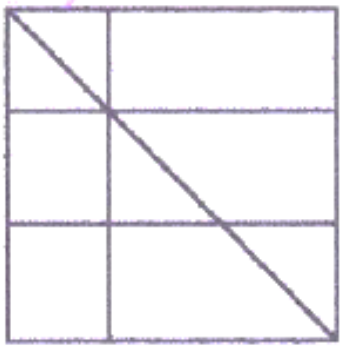
Fraction Puzzles **Name** _____

Write the fraction of the whole square that each section represents.

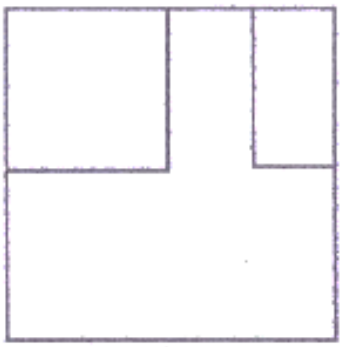


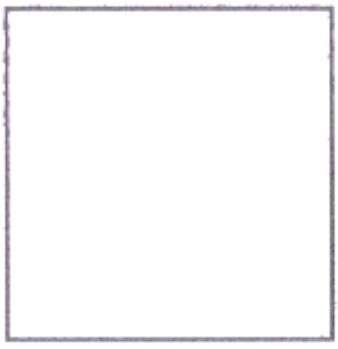






Make your own puzzle:





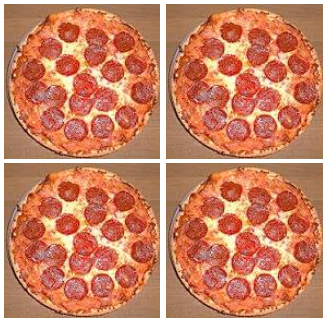
Mixed Numbers **Name** _____

Draw lines if needed and shade the figure to model the mixed number. Then write the equivalent improper fraction.

1. $1\frac{1}{3} =$

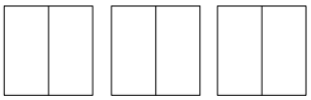


2. $3\frac{1}{8} =$

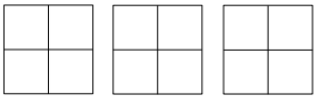


"Pizza Anyone" is licensed under CC BY-NC 4.0 / A derivative from the original work.

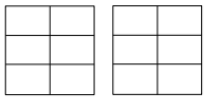
3. $2\frac{1}{2} =$



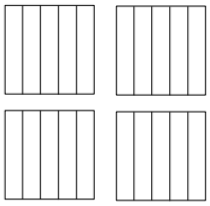
4. $2\frac{3}{4} =$



5. $1\frac{5}{6} =$



6. $3\frac{1}{5} =$



Summary: To convert a mixed number to an improper fraction, _____ the whole number part by the _____ and then _____ the _____. Write this number as the new _____ and keep the same _____.

Draw lines and shade the figure to model the improper fraction. Then write the improper fraction as a mixed number.

7. $\frac{5}{4} =$



8. $\frac{11}{6} =$



9. $\frac{12}{5} =$



Summary: To convert an improper fraction to a mixed number, _____ the numerator by the _____ to get the whole number part. Write the remainder as the _____ of the fractional part and keep the same _____.

Convert the mixed number to an improper fraction, then perform the indicated operation. Write your answer both as an improper fraction and as a mixed number.

10. $1\frac{2}{5}+\frac{4}{5}$

11. $\frac{2}{3}-4\frac{1}{2}$

12. $-2\frac{1}{3}\cdot\frac{4}{5}$

13. $2\frac{1}{4}\div\frac{5}{6}$

Mixed Operations with Fractions **Name** _____

Review: Discuss with your group and write a rule or procedure for each operation with fractions.

Addition:

Subtraction:

Multiplication:

Division:

Perform each operation specified and reduce your answer to simplest terms.

1. $\frac{2}{3}+\frac{3}{4}$

2. $\frac{2}{3}-\frac{3}{4}$

3. $\frac{2}{3}\cdot\frac{3}{4}$

4. $\frac{2}{3}\div\frac{3}{4}$

5. $-\frac{1}{6}-\frac{3}{12}$

6. $\left(\frac{2}{3}\right)^2$

7. $-\frac{7}{8} \div \frac{1}{4}$

8. $\frac{9}{16} \cdot \frac{4}{3} \cdot \frac{2}{5}$

9. $\frac{1}{2} - \frac{3}{4} + \frac{5}{8}$

10. $-2\frac{3}{8} + \frac{11}{6}$

11. $-6 \cdot \left(\frac{1}{3}\right)$

12. $1\frac{1}{2} \div (-3)$

13. Jamie walks $\frac{3}{4}$ of a mile to get on the bus and then $\frac{2}{5}$ of a mile from the bus stop to the store. To go to the store and back home, how many miles does Jamie walk? Show all of your steps and write your answer in a complete sentence.

14. Carlos is making Polvorones, which are Mexican Wedding Cookies. The recipe calls for $1\frac{1}{4}$ cups of butter. If the recipe makes five dozen cookies, how much butter is in one cookie? (Bonus if you can convert the answer to tablespoons or teaspoons.)

Order of Operations with Fractions **Name:** _____

Perform the following using the order of operations. Work slowly and carefully with your group members to make sure you are using the order of operations appropriately and completing computations correctly.

1. $\frac{3}{4}-\frac{1}{2}\cdot\frac{2}{3}$

2. $\frac{1}{2}\div\frac{1}{2}\cdot\frac{4}{5}$

3. $\frac{7}{15}+\frac{3}{5}\left(\frac{2}{6}\right)$

4. $\left(\frac{2}{3}\div\frac{16}{9}\right)-3\left(\frac{1}{15}\right)$

5. $\frac{1}{6}+\frac{11}{4}\left(-\frac{2}{3}\right)^2$

Challenge Problem:

6. $\left(\frac{11}{5}-1\frac{2}{3}\right)-\left(-\frac{4}{9}\cdot 18\right)^2$