MTH 112: Identities and Formulas Reference Sheet

This reference sheet may be provided to students during exams and other assessments.
Items may be removed with instructor discretion; however, nothing may be added.

Law of Sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

Double Angle Identities

$$\sin(2A) = 2\sin(A)\cos(A)$$
$$\cos(2A) = 1 - 2\sin^2(A)$$
$$\cos(2A) = 2\cos^2(A) - 1$$
$$\cos(2A) = \cos^2(A) - \sin^2(A)$$
$$\tan(2A) = \frac{2\tan(A)}{1-\tan^2(A)}$$

Sum and Difference Identities

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$
$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$$
$$\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$$

Half Angle Identities

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{2}}$$
$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos(A)}{2}}$$
$$\tan\left(\frac{A}{2}\right) = \frac{1-\cos(A)}{\sin(A)}$$

Product-to-Sum Identities

$$\sin(A)\sin(B) = \tfrac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$
$$\cos(A)\cos(B) = \tfrac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$
$$\sin(A)\cos(B) = \tfrac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$

Dot Product

$$\vec{v}\cdot\vec{w} = v_1 w_1 + v_2 w_2$$

Angle Between Vectors

$$\cos(\theta) = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|}$$

Euler's Formula

$$re^{i\cdot\theta} = r\cos(\theta) + r\sin(\theta)\cdot i$$

Sum-to-Product Identities

$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin(A) - \sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Conic Sections: Ellipses

Implicit Equation: $1 = \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2}$

Parametric System: $\begin{cases} x = a\cos(t) + h \\ y = b\sin(t) + k \end{cases}$

Sum of Sine and Cosine Identity

$$A_1\sin(\omega t) + A_2\cos(\omega t) = A\sin(\omega t + \phi)$$

where $A = \sqrt{A_1^{\,2} + A_2^{\,2}}$ and $\tan(\phi) = \frac{A_2}{A_1}$, and

ϕ satisfies $\cos(\phi) = \frac{A_1}{A}$ and $\sin(\phi) = \frac{A_2}{A}$.