MTH 112 Lab Manual v1.3 12/18/2025

Prepared by
Portland Community College
Mathematics Department

Contents

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This is the second part of the two part lab manual for MTH 111 and MTH 112 at Portland Community College. The first chapter is "Chapter 5" to align with the first portion of the lab manual.

This lab manual was designed with the intent of being used in the following manner. In each section:

- Students will complete the Preparation Exercises before receiving instruction on the content. Some instructors will have students do this before coming to class, while others might do it as a warmup in class.
- There will be some instructor-led presentation of the content. This may be a formal class lecture, a discovery activity, a video lecture, or something else.
- Students will then engage in Practice Exercises in a group setting to reinforce their initial understanding of the foundational concepts.
- An instructor will then assign one or more group activities. Because many of these activities are web-based and instructors can choose to use different activities, we have not included any of the possible activities in this document.
- The Definitions are meant to provide a single location for all definitions and some key concepts. Students can use this as a resource after having covered the topics in class.
- Students will complete some or all of the Exit Exercises, as decided by their instructor, to summarize their understanding of key concepts.
- In general, not every section of this lab manual will take the same amount of class time. Some sections might be half-day topics, while others could be two-day topics.
- Each instructor will identify for their class which exercises will be submitted as part of the course grade.

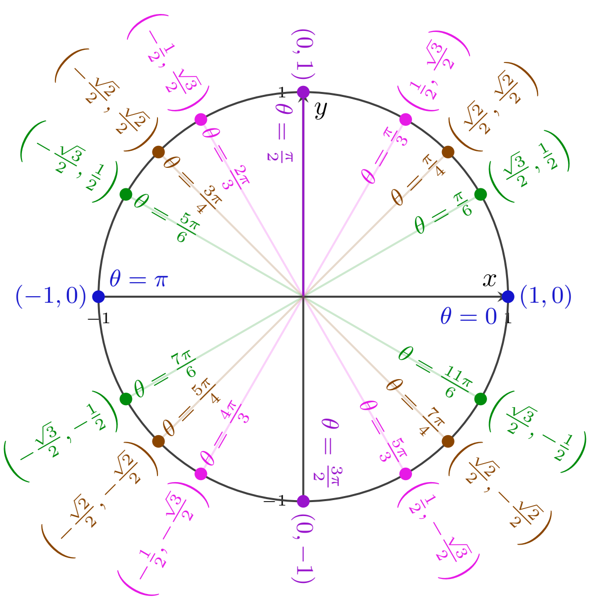
Course Resource Links:
Algebra and Trigonometry 2e: http://tiny.cc/112Z-Textbook
MTH 112 Supplement: http://tiny.cc/112Z-Supplement

Chapter 5

The Unit Circle

Contents

Figure 5.1: Unit Circle with Standard Values



5.1 Angles

In this section we will draw angles in standard position, convert between degrees and radians and degrees-minutes-seconds, find coterminal angles, and find the length of a circular arc.

Textbook Reference: This relates to content in §7.1 of *Algebra and Trigonometry 2e*.

Preparation Exercises

Answer the following without using a calculator. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

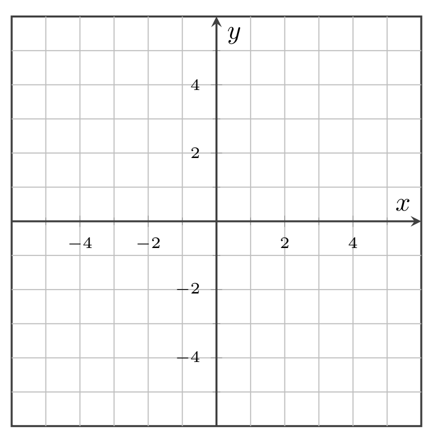
Preparation 1:
Solve the equation $T = \frac{v}{w^2}$ for w.

Preparation 2:
Simplify the expression $80 \cdot \frac{\pi}{180}$ without a calculator. Reduce to simplest form.

Preparation 3:
In terms of time, how many minutes total are in 3 hours and 53 minutes?

Preparation 4:
Draw a line segment from the point $(5, -3)$ to the point $(-1, 2)$. Find the distance between those two points using the Pythagorean Theorem. Leave your answer in exact form.

Figure 5.2: A grid to plot your points



Practice Exercises

Practice 1:
Find an angle that is coterminal to $-40°$ and is between $360°$ and $720°$.

Practice 2:
Match the pairs of coterminal angles.

- $30°$
- $270°$
- $-220°$
- $-60°$
- $0°$
- $150°$

- $720°$
- $-90°$
- $390°$
- $510°$
- $-420°$
- $500°$

Practice 3:
Convert the angles.

a) Convert $28°$ to radians. Leave your answer as a fraction in terms of π.

b) Convert $\frac{7\pi}{9}$ radians to degrees. Round to 6 digits behind the decimal place.

c) Convert $14°20'36''$ to a decimal measurement of degrees.

d) Convert $76.85°$ to degrees-minutes-seconds.

Practice 4:

Use the formula $s = r\theta$ to find the missing values.

a) Find the arc length of on a circle of radius 6 with an interior angle of $\frac{4\pi}{7}$.

b) Find the interior angle of an arc length on a circle of radius 12 with an arc length of $\frac{3\pi}{2}$.

c) Find the arc length of on a circle of radius 10π with an interior angle of $20°$.

Definitions

Angle

An **angle** is formed when two lines intersect at a point. We can measure the "size" of this angle in degrees or radians.

Ray

A **ray** is a portion of a line that starts at a point and extends infinitely in one direction.

Degree

A **degree** is an angle measurement equal to $\frac{1}{360}$ of a full rotation.

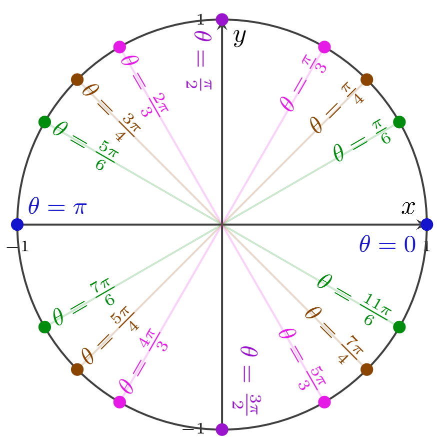
Right Angle

A **right angle** is an angle of $90°$.

Radian

A **radian** is a unit of measurement of angle size, and 1 radian is defined to be the angle that is made when the arc length for that angle on a circle of radius 1 is equal to 1. Check out this Desmos link and click the "play" button on the slider for a visual representation of this concept.

Figure 5.3: Standard Angles



Initial and Terminal sides

An angle is created by imagining two rays originating from the same point: the **initial side** of the angle is stationary and the **terminal side** is rotated around until the desired angle is created. Angles measured from initial to terminal side in a counter-clockwise direction are positive. Angles measured from initial to terminal side clockwise are negative.

Standard Position

An angle is said to be in **standard position** if the initial and terminal sides of the angle meet at the origin and the initial side of the angle is along the positive x-axis.

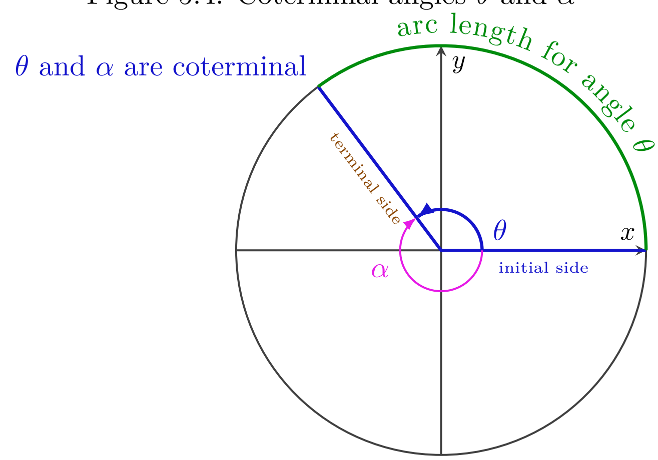
Coterminal

Two angles are **coterminal** if their initial and terminal sides align, but they differ by an integer number of full rotations. In standard position, this means that the two angles are "in the same place" are different by some integer multiple of $360°$ (or 2π radians).

Arc length

An **arc length** is a distance along a part of the circumference of a circle. A formula for arc length is $s = r\theta$, where s stands for arc length along a circle of radius r, with an interior angle θ centered at the center of the circle.

Figure 5.4: Coterminal angles θ and α



Exit Exercises

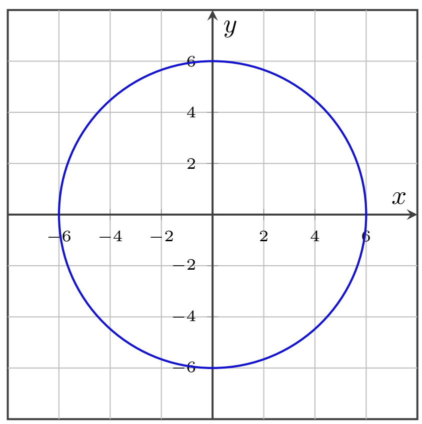
Exit 1:
Explain why $-90°$ is not *the same* as $270°$.

Exit 2:
Convert $34°29'24''$ to decimal degrees.

Exit 3:
Convert the angle $144°$ to radians. Write you answer in exact form as a fraction with π in it.

Exit 4:
How big of an arc length will an angle of $\frac{3\pi}{8}$ radians create on a circle of radius 6? Draw this angle and arc length on the circle of radius 6 provided.

Figure 5.5: A circle of radius 6



On a scale of 1 – 5, how are you feeling with the concepts related to angles?
Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.

1 2 3 4 5

5.2 Right Triangle Trigonometry

In this section we will use right triangles to evaluate trigonometric functions, find function values for 30°, 45°, and 60°, use all six trigonometric functions to find lengths inside right triangles, and use right-triangle trigonometry to solve applied problems.

Textbook Reference: This relates to content in §7.2 of *Algebra and Trigonometry 2e*.

Preparation Exercises

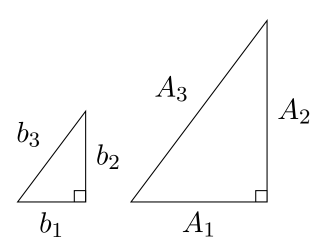
Answer the following without using a calculator. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

Preparation 1:
What does the word "adjacent" mean?

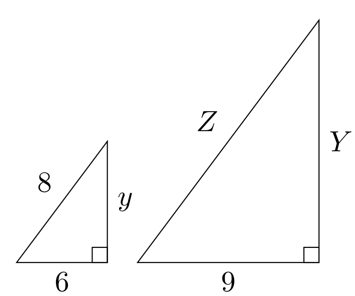
Preparation 2:
Fill in the table with the missing angle measurements.

Angle (radians)	$\frac{\pi}{4}$			$\frac{2\pi}{3}$	$\frac{5\pi}{6}$		π	
Angle (degrees)		30°	90°			60°		135°

Preparation 3:
Recall that "similar triangles" are two triangles with equal angle measurements, but not necessarily equal side lengths. The ratio of any two corresponding sides of the triangles will be equal, which means that $\frac{A_1}{b_1} = \frac{A_2}{b_2} = \frac{A_3}{b_3}$.



Using the similar triangles equations, solve for the missing sides. Drawing may not be drawn to scale.



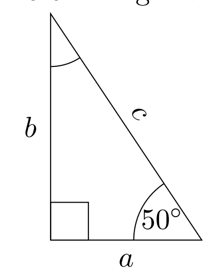
Practice Exercises

Practice 1:
Decide which values match without a calculator using "SOHCAHTOA" and "CHOSHACAO". The triangles are not shown to scale.

a) Which of the values will be the same as $\sin(40°)$?

- $\sin(50°)$
- $\cos(50°)$
- $\tan(40°)$
- $\cos(40°)$

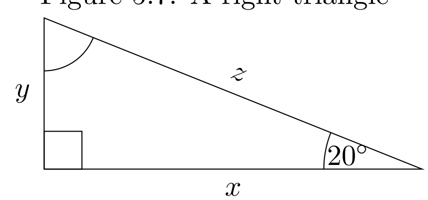
Figure 5.6: A right triangle



b) Which of the values will be the same as $\tan(20°)$?

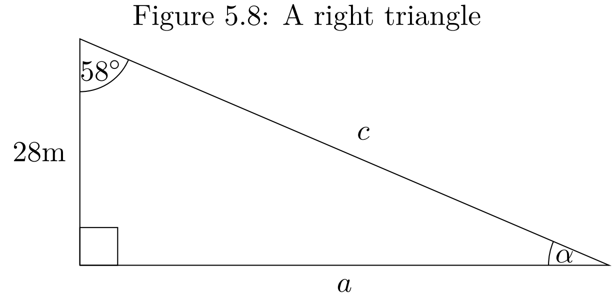
- $\cot(20°)$
- $\tan(70°)$
- $\cot(70°)$
- $\cos(70°)$

Figure 5.7: A right triangle



Practice 2:
For the triangle shown in Figure 5.8, find a, c, and α. You can use a calculator to help get an approximation, accurate to 4 digits behind the decimal place.

Figure 5.8: A right triangle



Practice 3:

On a backpacking trip in the Wallowa Mountains of northeastern Oregon, Megan and Emily were marveling at the height of an enormous Ponderosa Pine. Ross said, "we can calculate it's height pretty easily just by knowing that 'a pace' is 3ft and the distance from the tip of my thumb to the tip of my pinkie is 9 inches". Ross paced away from the tree on level ground 30 paces. Help the three hikers measure the height of the tree two different ways.

a) Ross finds a stick and measures it to be about 6ft tall. He lays down on the ground and asks Emily to hold the stick vertically on the ground at a certain point where the tip of the stick lines up with the tip of the tree from his point of view. That distance on the ground was 4ft from Ross's viewing position, as shown in Figure 5.9. Set up similar triangles to estimate the height of the tree. Drawing not to scale.

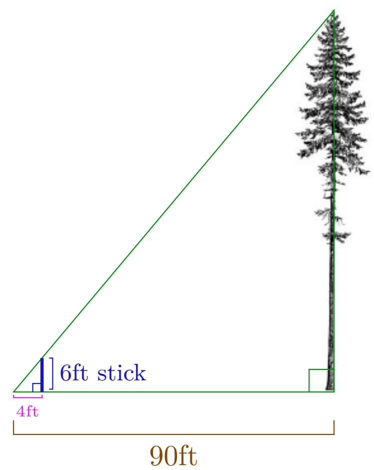


Figure 5.9: A tree in the woods

b) Ross then took out his phone and used the inclinometer app on to measure the angle to to top of the tree, to be about 55°. In this scenario, Ross still 30 paces away from the tree, and was standing to measure the height: the phone was 5ft above the ground, as shown in Figure 5.10. Use right-triangle trigonometry to estimate the height of the tree. Drawing not to scale.

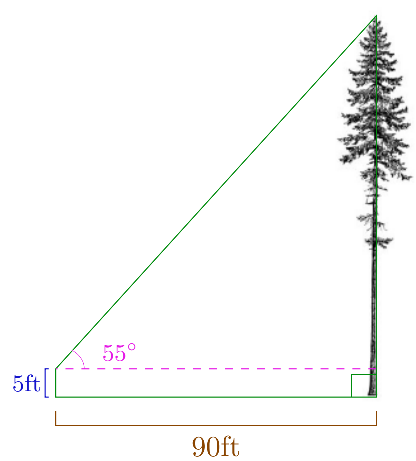


Figure 5.10: A tree in the woods

Definitions

Consider the triangle shown in Figure 5.11.

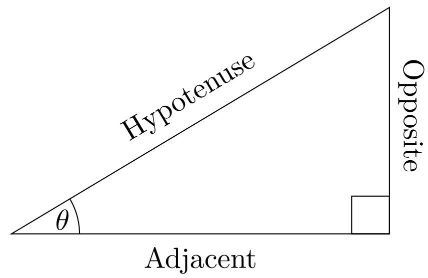


Figure 5.11: A right triangle

SOHCAHTOA

"SOHCAHTOA" stands for

- **S**ine is **O**pposite over **H**ypotenuse
- **C**osine is **A**djacent over **H**ypotenuse
- **T**angent is **O**pposite over **A**djacent

These mean the following:

- $\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$
- $\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$
- $\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$

CHOSHACAO

"CHOSHACAO" stands for

- **C**osecant is **H**ypotenuse over **O**pposite
- **S**ecant is **H**ypotenuse over **A**djacent
- **C**otangent is **A**djacent over **O**pposite

These mean the following:

- $\csc(\theta) = \frac{\text{Hypotenuse}}{\text{Opposite}}$
- $\sec(\theta) = \frac{\text{Hypotenuse}}{\text{Adjacent}}$
- $\cot(\theta) = \frac{\text{Adjacent}}{\text{Opposite}}$

Exit Exercises

Exit 1:

Use the triangle in Figure 5.12 to answer the questions below.

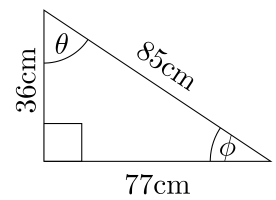


Figure 5.12: A right triangle

(i) Find the value of $\cos(\theta)$.

(ii) Find the value of $\tan(\phi)$.

(iii) Find the value of $\csc(\theta)$.

Exit 2:

Shown in Figure 5.13 is a triangle with side $b = 11$, angle $B = 22°$, and angle $C = 90°$. Find the length of the side a.

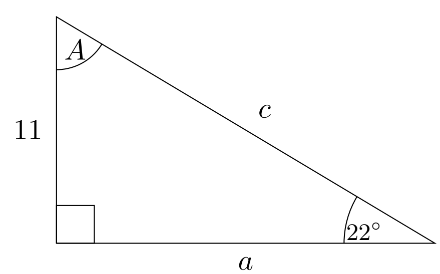
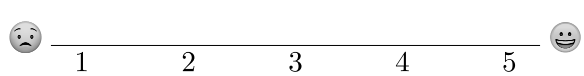


Figure 5.13: A right triangle

On a scale of 1 – 5, how are you feeling with the concepts related to right triangle trigonometry? Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.



5.3 The Unit Circle

In this section we will find function values for the sine and cosine of 30°, 45°, and 60°, identify the domain and range of sine and cosine functions, find reference angles, and use reference angles to evaluate trigonometric functions.

Textbook Reference: This relates to content in §7.3 of *Algebra and Trigonometry 2e*.

Preparation Exercises

Answer the following without using a calculator. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

Preparation 1:
For a right triangle with side $a = \frac{1}{2}$ and hypotenuse $c = 1$, find the missing side, b, using the Pythagorean Theorem.

Preparation 2:
Which quadrant are the following angles in?

(i) $\frac{3\pi}{4}$

(ii) $\frac{4\pi}{3}$

(iii) $\frac{11\pi}{6}$

(iv) $\frac{\pi}{3}$

(v) $-\frac{\pi}{6}$

(vi) $-\frac{2\pi}{3}$

(vii) $-\frac{7\pi}{6}$

(viii) $\frac{5\pi}{6}$

Preparation 3:
Simplify $2\pi - \frac{2\pi}{3}$ without a calculator.

Preparation 4:
Count from 0 to 2π by multiples of $\frac{\pi}{12}$, and then reduce those fractions. Here's how to start out: $\frac{0\pi}{12}$, $\frac{\pi}{12}$, $\frac{2\pi}{12}$, $\frac{3\pi}{12}$, . . .

Practice Exercises

Practice 1:

a) If $\cos(\theta) = \frac{1}{5}$ and $\sin(\theta) = -\frac{2\sqrt{5}}{5}$, which quadrant must θ be in?

b) If $\cos(\phi) = -\frac{2}{3}$ and $\sin(\phi) = \frac{\sqrt{5}}{3}$, which quadrant must ϕ be in?

Practice 2:

Which of the standard angles between 0 and 2π on the unit circle do the following x and y coordinates go with?

a) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ b) $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ c) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Practice 3:

Find the reference angles for the given angles.

a) $340°$ b) $440°$ c) $\frac{8\pi}{3}$ d) $\frac{9\pi}{7}$

Practice 4:

a) If $\cos(\theta) = \frac{1}{5}$ and $0 \leq \theta \leq \frac{\pi}{2}$, find the value of $\sin(\theta)$.

b) If $\sin(\alpha) = -\frac{3}{7}$ and $\pi \leq \alpha \leq \frac{3\pi}{2}$, find the value of $\cos(\alpha)$.

c) If $\cos(\beta) = -\frac{3}{4}$ and $\frac{\pi}{2} \leq \beta \leq \pi$, find the value of $\sin(\beta)$.

Practice 5:

Evaluate the expressions without a calculator by first finding and using the reference angles for the angles shown.

a) $\sin(\frac{3\pi}{2})$

b) $\sin(\frac{7\pi}{3})$

c) $\cos(\frac{11\pi}{4})$

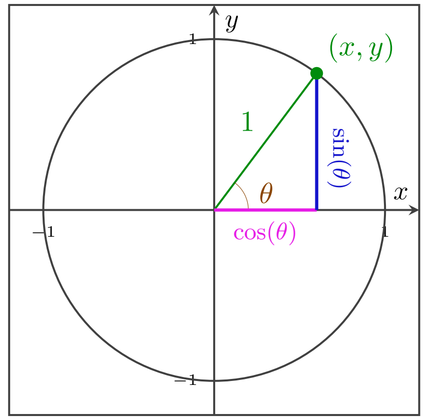
d) $\cos(\frac{15\pi}{6})$

Definitions

The Unit Circle

A **unit circle** is a circle with radius 1.

Figure 5.14: Unit Circle



Cosine of an Angle

The **cosine of an angle**, θ, is equal to the x-value at that angle on a unit circle.

Sine of and Angle

The **sine of an angle**, θ, is equal to the y-value at that angle on a unit circle.

The Pythagorean Identity

The **Pythagorean Identity** is the relationship between sine and cosine values, $\sin^2(\theta) + \cos^2(\theta) = 1$, relating to the Pythagorean Theorem on Figure 5.14.

Reference Angle

A **reference angle**, α, to an angle θ, is the acute (or right) positive angle between the x-axis to the angle θ, as shown in Figure 5.15.

Figure 5.15: Reference Angles

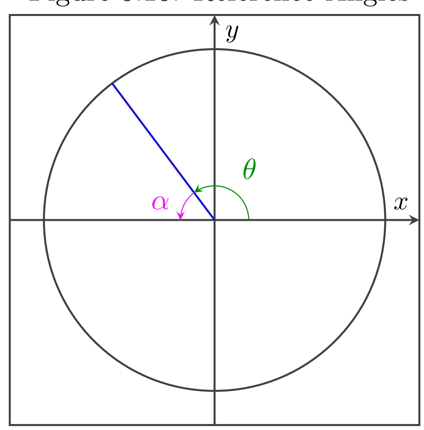
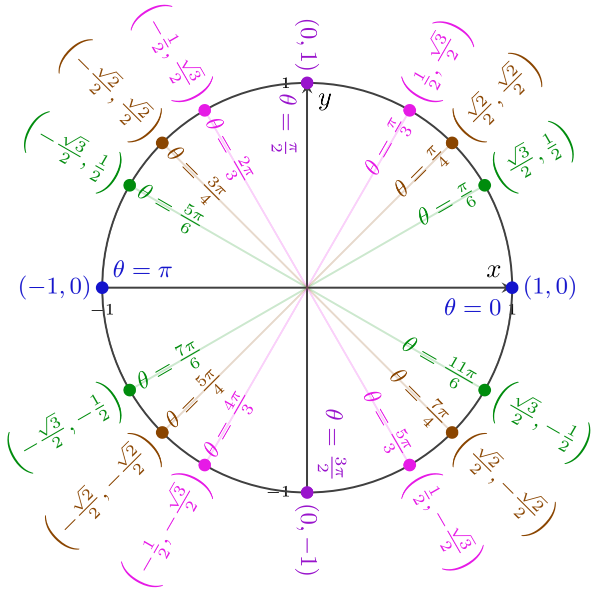


Figure 5.16: Unit Circle with Standard Values



Exit Exercises

Exit 1:
Which angle, θ, between 0 and 2π has a sine value of $-\frac{\sqrt{3}}{2}$ and a cosine value of $\frac{1}{2}$?

Exit 2:
What is the reference angle for $\frac{7\pi}{6}$?

Exit 3:
Evaluate $\sin(\frac{11\pi}{3})$ without a calculator.

Exit 4:
If $\sin(\theta) = -\frac{1}{4}$ and $\pi \leq \theta \leq \frac{3\pi}{2}$, find the value of $\cos(\theta)$.

On a scale of 1 – 5, how are you feeling with the concepts related to the unit circle?
Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.

1 2 3 4 5

5.4 Other Trigonometric Functions

In this section, we will find exact values of the trigonometric functions secant, cosecant, tangent, and cotangent of $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, use reference angles to evaluate the trigonometric functions secant, tangent, and cotangent, use properties of even and odd trigonometric functions, recognize and use fundamental identities, and evaluate trigonometric functions with a calculator.

Textbook Reference: This relates to content in §7.4 of *Algebra and Trigonometry 2e.*

Preparation Exercises

Answer the following without using a calculator. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

Preparation 1:
What did "CHOSHACAO" stand for back in Exercise 11 from Section 5.2?

Preparation 2:
What is the value of $\sin\left(\frac{\pi}{3}\right)$?

Preparation 3:
Find the value of $\cos\left(\frac{11\pi}{4}\right)$ using your memorized unit circle values and a reference angle.

Preparation 4:
Find the value of $\sin\left(-\frac{2\pi}{3}\right)$ using your memorized unit circle values and a reference angle.

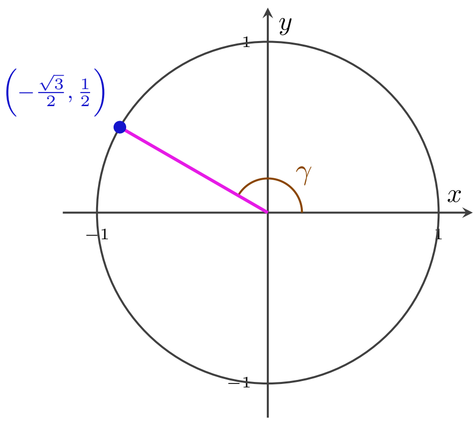
Preparation 5:
Explain what makes a function an "odd function"?

Practice Exercises

Practice 1:

Find all six trigonometric function values given the coordinates of the point at an angle γ on a unit circle.

Figure 5.17: A point on a unit circle at an angle γ



a) $\sin(\gamma)$

b) $\cos(\gamma)$

c) $\csc(\gamma)$

d) $\sec(\gamma)$

e) $\tan(\gamma)$

f) $\cot(\gamma)$

Practice 2:

Find all six trigonometric function values at the angle $\frac{\pi}{3}$ using your memorized values for the $\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{3}\right)$.

a) $\sin\left(\frac{\pi}{3}\right)$

b) $\cos\left(\frac{\pi}{3}\right)$

c) $\csc\left(\frac{\pi}{3}\right)$

d) $\sec\left(\frac{\pi}{3}\right)$

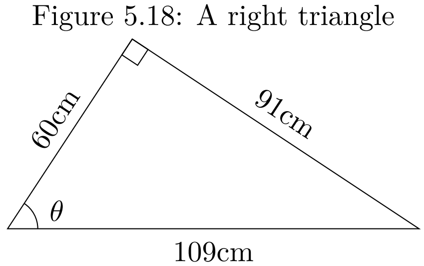
e) $\tan\left(\frac{\pi}{3}\right)$

f) $\cot\left(\frac{\pi}{3}\right)$

Practice 3:

Find the trigonometric function values given the right triangle with lengths shown in Figure 5.18.

Figure 5.18: A right triangle



a) $\sin(\theta)$

b) $\csc(\theta)$

c) $\sec(\theta)$

d) $\cot(\theta)$

Practice 4:
Fill in a table of all 6 trig values for the standard angles on the unit circle.

θ	0	$\frac{5\pi}{6}$	$\frac{\pi}{4}$	$\frac{4\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$					
$\cos(\theta)$					
$\csc(\theta)$					
$\sec(\theta)$					
$\tan(\theta)$					
$\cot(\theta)$					

Practice 5:
Use the fact that $\sin(x)$, $\tan(x)$, $\cot(x)$, and $\csc(x)$ are odd and $\cos(x)$ and $\sec(x)$ are even to fill in the following values.

a) $\sin\left(-\frac{2\pi}{3}\right)$

d) $\csc\left(-\frac{2\pi}{3}\right)$

b) $\tan\left(-\frac{2\pi}{3}\right)$

e) $\cos\left(-\frac{2\pi}{3}\right)$

c) $\cot\left(-\frac{2\pi}{3}\right)$

f) $\sec\left(-\frac{2\pi}{3}\right)$

Practice 6:
Find the other trigonometric values for the angle shown in each part.

a) $\sec(\psi) = -\frac{7}{2}$ and $\frac{\pi}{2} \leq \psi \leq \pi$.

b) $\cot(\zeta) = -\frac{6}{5}$ and $\frac{3\pi}{2} \leq \zeta \leq 2\pi$.

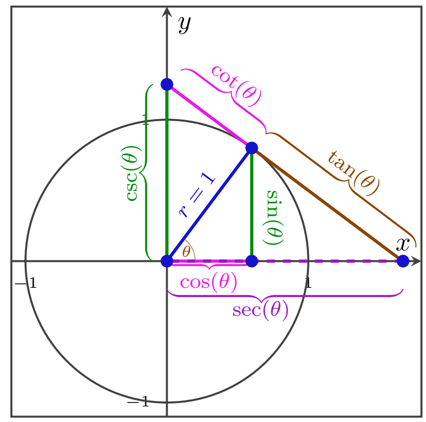
Definitions

The "Other" Trigonometric Functions

For a right triangle with with angle θ, the "**other trigonometric functions**" can be found by the following definitions as in Figure 5.19. Visit this interactive Desmos graph to see the six trigonometric lengths in action.

- $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$
- $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$
- $\sec(\theta) = \frac{1}{\cos(\theta)}$
- $\csc(\theta) = \frac{1}{\sin(\theta)}$

Figure 5.19: Other trigonometric function definitions

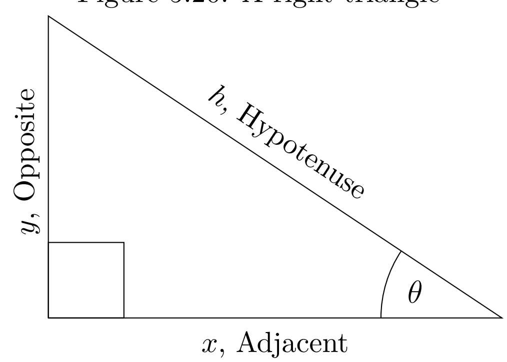


CHOSHACAO

For a right triangle with hypotenuse h, and sides x, and y, as in Figure 5.20, tangent is still "opposite over adjacent", as you saw in the section about right-triangle trigonometry and the "other trigonometric function values" can be found with "**CHOSHACAO**".

- $\tan(\theta) = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}$
- $\cot(\theta) = \frac{x}{y} = \frac{\text{Adjacent}}{\text{Opposite}}$
- $\sec(\theta) = \frac{h}{x} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$
- $\csc(\theta) = \frac{h}{y} = \frac{\text{Hypotenuse}}{\text{Opposite}}$

Figure 5.20: A right triangle



Even and Odd Trigonometric Functions

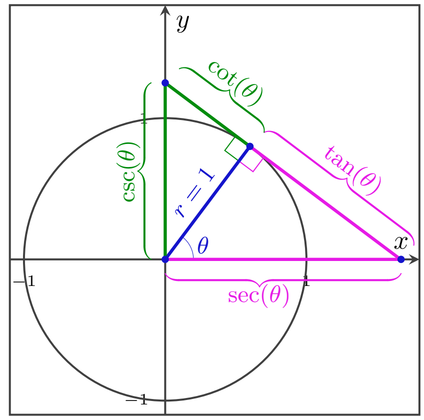
Recall that even functions are symmetrical about the y-axis, which makes $f(x) = f(-x)$, and odd functions are symmetrical about the origin, which makes $-f(x) = f(-x)$. All of the trigonometric functions are either even or odd. Here's a list:

- $\sin(-x) = -\sin(x)$, so $\sin(x)$ is odd.
- $\tan(-x) = -\tan(x)$, so $\tan(x)$ is odd.
- $\cot(-x) = -\cot(x)$, so $\cot(x)$ is odd.
- $\csc(-x) = -\csc(x)$, so $\csc(x)$ is odd.
- $\cos(-x) = \cos(x)$, so $\cos(x)$ is even.
- $\sec(-x) = \sec(x)$, so $\sec(x)$ is even.

Other Pythagorean identities

You already know that $\sin^2(\theta) + \cos^2(\theta) = 1$, and that this is called the **Pythagorean Identity**. There are two other Pythagorean relationships which you can see Figure 5.21, which is a trimmed-down version of Figure 5.19.

Figure 5.21: Other Pythagorean Identities



- $\tan^2(\theta) + 1 = \sec^2(\theta)$
- $\cot^2(\theta) + 1 = \csc^2(\theta)$

Periods of trigonometric functions

The **period** of a function is the length of the shortest x-interval over which a function completes one full cycle. This can be thought of as the shortest distance that a graph can be shifted left or right before completely aligning with the original graph. Mathematically, this would be that the period, P, of a repeating function f is the smallest value such that $f(x + P) = f(x)$, for any value of x.

- The period of $\sin(x)$, $\csc(x)$, $\cos(x)$, and $\sec(x)$ is 2π.
- The period of $\tan(x)$ and $\cot(x)$ is π.

Exit Exercises

Exit 1:

Evaluate $\sec\left(\frac{-3\pi}{4}\right)$.

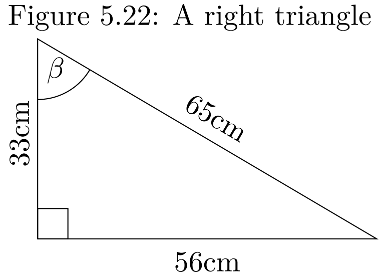
Exit 2:

If $\cot(\alpha) = -\frac{2}{7}$ and $\frac{\pi}{2} \leq \alpha \leq \pi$, find the value of $\tan(\alpha)$ and $\sec(\alpha)$.

Exit 3:

Find the trigonometric values given the right triangle with lengths shown.

Figure 5.22: A right triangle



a) $\csc(\beta)$

b) $\cot(\beta)$

c) $\cos(\beta)$

On a scale of 1 – 5, how are you feeling with the concepts related to the “other” trigonometric functions? Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.

1 2 3 4 5

Chapter 6

Periodic Functions

Contents

6.1 Graphs of Sine and Cosine Functions

In this section, we will graph transformations of $y = \sin(x)$ and $y = \cos(x)$ and examine phase shifts of sine and cosine curves.

Textbook Reference: This relates to content in §8.1 of *Algebra and Trigonometry 2e.*

Preparation Exercises

Answer the following without using a calculator. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

Preparation 1:

a) What is the period of the sine function? Of the cosine function?

b) What is the range of the sine function? Of the cosine function?

Preparation 2:

Imagine a function $y = h(x)$, with some points given in the table below. Fill in the table to show how the points are transformed for the given function values.

Points on $y = h(x)$	$(0, 0)$	$(1, 1)$	$(-1, -1)$	$(-3, 2)$	$(2, -3)$
Points on $y = 3h(x)$					
Points on $y = 3h(2x)$					
Points on $y = 3h(2(x-1))$					
Points on $y = 3h(2(x-1)) + 4$					

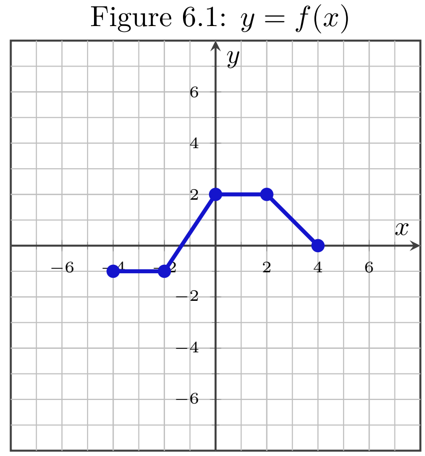
Preparation 3:

Make the following graphs on Figure 6.1 given the graph of f shown in Figure 6.1.

a) Make a graph of $y = -2f(x) - 3$.

b) Make a graph of $y = -3f(2(x+1)) - 1$.

Figure 6.1: $y = f(x)$



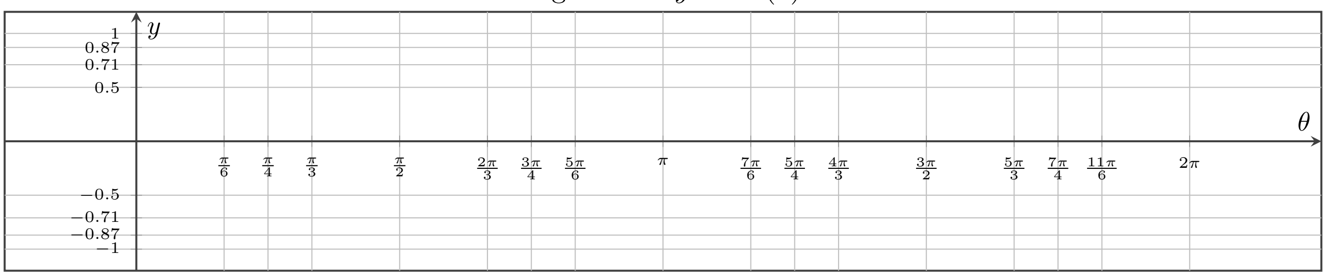
Practice Exercises

Practice 1:

Make a graph of $y = \sin(x)$ by plotting the points that you know from the table. You need to know that $\frac{\sqrt{2}}{2} \approx 0.71$ and $\frac{\sqrt{3}}{2} \approx 0.87$.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0

Figure 6.2: $y = \sin(\theta)$



Practice 2:

Comparing to $y = \sin(x)$...

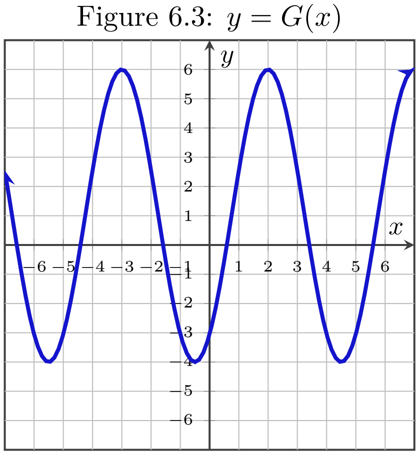
a) What is the phase shift for the function $2\sin(3x - \pi) + 4$? Interpret this as a fraction of a period shift.

b) What is the horizontal shift for the function $2\sin(3x - \pi) + 4$?

Practice 3:

For the graph of G shown in Figure 6.3, answer the following questions.

Figure 6.3: $y = G(x)$



a) What is the range of G?

b) What is the midline, $y = D$, of G?

c) What is the amplitude, A of G?

d) What is the period, P, of G?

e) What is the value of B for G?

Still considering the graph of G shown in Figure 6.3...

f) If we think of G as a transformation of $\sin(x)$, how much of a horizontal shift would have happened to create G?

g) Write a formula for G as a transformation of $\sin(x)$, $G(x) = A\sin(B(x - C)) + D$.

h) If we think of G as a transformation of $\cos(x)$, how much of a horizontal shift would have happened to create G?

i) Write a formula for G as a transformation of $\cos(x)$, $G(x) = A\cos(B(x - C)) + D$.

Practice 4:
Answer the questions about the function $T(x) = 3\sin\left(\frac{\pi}{3}(x + 1)\right) - 5$.

a) What is the midline of the function T?

b) What is the amplitude, A, of the function T?

c) What is the range of the function T?

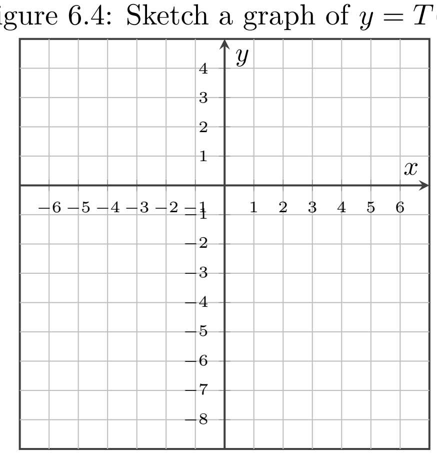
d) What is the value of B for the function T?

e) What is the period, P, of the function T?

f) What does the "$x + 1$" portion of the function do as a transformation of $\sin(x)$?

g) Make a graph of T onto Figure 6.4 with the what you know about the shape of $\sin(x)$ and the information that you gained in the above parts. Check your result on a calculator *after* you've tried everything by hand.

Figure 6.4: Sketch a graph of $y = T(x)$



Definitions

Even/Odd Function Properties of Sine and Cosine

Sine is an odd function, which means that it is symmetrical over the origin. Cosine is an even function, which means that it is symmetrical over the y-axis.

Period

The **period** of a function is the smallest number, P, such that a horizontal shift of P units results in a graph that perfectly overlaps the graph of the original function. Try this Desmos link to see a visual on the topic.

Midline

The **midline** of a periodic function is the horizontal line, $y = D$, that is exactly halfway between the highest and lowest y-values on the graph. You can find this equation by averaging the highest and lowest y-values on the graph with the formula $y = \frac{\text{highest } y\text{-value} + \text{lowest } y\text{-value}}{2}$.

Amplitude

The **amplitude** of a periodic function is the vertical distance from the midline to the highest y-value on the graph or the vertical distance from the midline to the lowest y-value on the graph. You can find this distance with the formula $|A| = \frac{\text{highest } y\text{-value} - \text{lowest } y\text{-value}}{2}$.

Sinusoidal Function

A **sinusoidal function** is a transformation of a sine or cosine function. Sinusoidal functions have the form $f(x) = A\sin(B(x - C)) + D$ or $f(x) = A\cos(B(x - C)) + D$.

Phase Shift

The **phase shift** of a periodic function represents the fraction of a period that the graph has been shifted from the base function. You can find the phase shift of a sinusoidal function ($f(x) = A\sin(B(x - C)) + D$ or $f(x) = A\cos(B(x - C)) + D$) by evaluating $\frac{BC}{2\pi}$. Please note that this definition is different from the one that the book gives, but is consistent with most other physics, engineering, and technical mathematics texts. The book defines the **phase shift** to be the same as the horizontal shift, for which we already have a name.

Horizontal Shift

A sinusoidal function, $f(x) = A\sin(B(x - C)) + D$ or $f(x) = A\cos(B(x - C)) + D$, is **horizontally shifted** C units from the base function position.

Relationship between the B-value and the Period

The B value of a sinusoidal function, $f(x) = A\sin(B(x - C)) + D$ or $f(x) = A\cos(B(x - C)) + D$, has a relationship with the period, P, of the function. Since the period is a positive number, we will use absolute values on B, since B might be negative. That relationship is that $|B| = \frac{2\pi}{P}$, which is the same as saying $P = \frac{2\pi}{|B|}$.

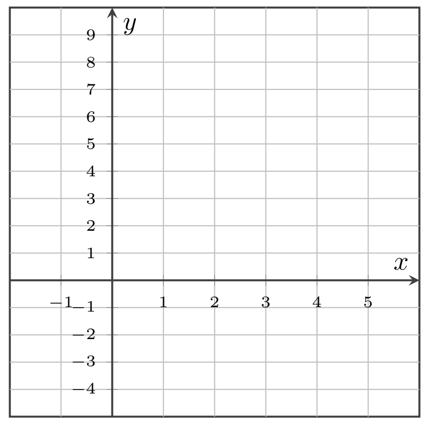
Exit Exercises

Exit 1:

Consider the function $W(x) = 5\sin\left(\frac{4\pi}{5}(x-1)\right) + 3$.

a) What is the period of the function $W(x)$?

Figure 6.5: Sketch a graph of $y = W(x)$



b) What is the midline of the function $W(x)$?

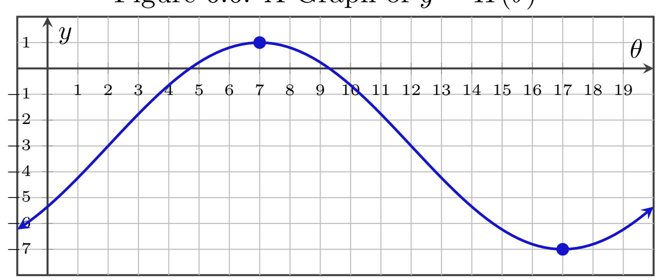
c) What is the amplitude of the function $W(x)$?

d) By hand, make a graph of the function $W(x)$ using the information above onto Figure 6.5.

Exit 2:

Find two possible formulas, one using sine and the other using cosine, for the graph shown in Figure 6.6.

Figure 6.6: A Graph of $y = H(\theta)$



On a scale of 1 – 5, how are you feeling with the concepts related to the graphs of sine and cosine? Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.

1 2 3 4 5

6.2 Graphs of Other Trigonometric Functions

In this section, we will discuss the graphs of tangent, cosecant, and secant and their domains.

Textbook Reference: This relates to content in §8.2 of *Algebra and Trigonometry 2e*.

Preparation Exercises

Answer the following without using a calculator. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

Preparation 1:
Evaluate $\tan\left(\frac{2\pi}{3}\right)$ using the definition of tangent: $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$.

Preparation 2:
Evaluate $\csc\left(-\frac{5\pi}{6}\right)$ using the definition of cosecant: $\csc(\theta) = \frac{1}{\sin(\theta)}$.

Preparation 3:
Which of the six trigonometric functions (sine, cosine, tangent, cosecant, secant, and cotangent) are odd functions?

Preparation 4:
What is the domain of the sine function? What is the domain of the cosine functions?

Preparation 5:
Which values of θ make it so that $\sin(\theta)$ is 0? Hint: there are infinitely many answers.

Practice Exercises

Recall that for a right triangle with with angle θ, the "other trigonometric functions" can be found by the following definitions. Visit this interactive Desmos graph to see the six trigonometric lengths in action.

- $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$
- $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$
- $\sec(\theta) = \frac{1}{\cos(\theta)}$
- $\csc(\theta) = \frac{1}{\sin(\theta)}$

Practice 1:

a) The cosecant function, $\csc(\theta) = \frac{1}{\sin(\theta)}$, would be undefined when $\sin(\theta) = 0$. When is $\sin(\theta) = 0$? There are infinitely many answers: write your answers in a full sentence describing the pattern.

b) Write your solutions to $\sin(\theta) = 0$ in set-builder notation.

c) Based on your answers to the above questions, what is the domain of the cosecant function? Write your answer in a sentence explaining the pattern.

Practice 2:

a) The secant function, $\sec(\theta) = \frac{1}{\cos(\theta)}$, would be undefined when $\cos(\theta) = 0$. When is $\cos(\theta) = 0$? There are infinitely many answers: write your answers in a full sentence describing the pattern.

b) Write your solutions to $\cos(\theta) = 0$ in set-builder notation.

c) Based on your answers to the above questions, what is the domain of the secant function? Write your answer in a sentence explaining the pattern.

Practice 3:

a) The tangent function, $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$, would be undefined when $\cos(\theta) = 0$. This is the same problem that the secant function had. So, what is the domain of the tangent function? Write your answer in a sentence explaining the pattern.

b) The cotangent function, $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$, would be undefined when $\sin(\theta) = 0$. This is the same problem that the cosecant function had. So, what is the domain of the cotangent function? Write your answer in a sentence explaining the pattern.

Practice 4:

Match the "other trigonometric" functions with their graphs, shown in Figures 6.7, 6.8, 6.9, and 6.10. Use the definitions of the functions and their domains to decide which goes with which. Check your answers with your favorite graphing program.

a) $y = \tan(x)$ b) $y = \cot(x)$ c) $y = \sec(x)$ d) $y = \csc(x)$

Figure 6.7: Graph A

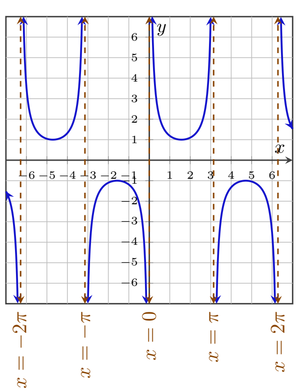


Figure 6.8: Graph B

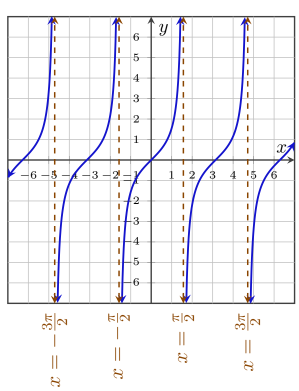


Figure 6.9: Graph C

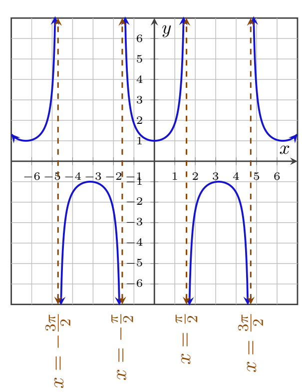
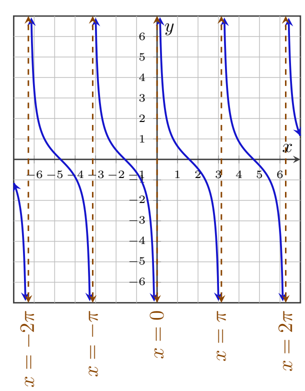


Figure 6.10: Graph D



Exit Exercises

Exit 1:

Which trigonometric functions have a domain that is the set of all real numbers except integer multiples of π?

Exit 2:

Which trigonometric function is defined to be $\frac{\cos(\theta)}{\sin(\theta)}$?

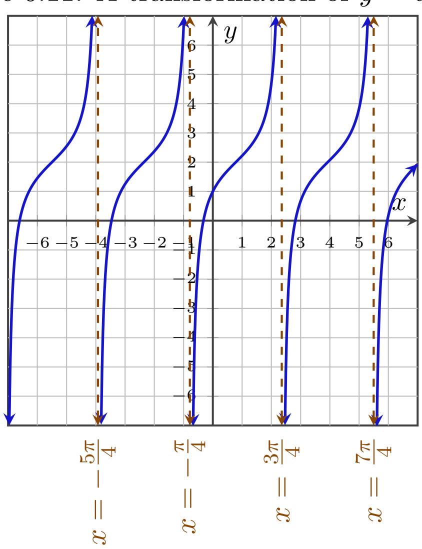
Exit 3:

Which trigonometric functions have a domain that is the set of all real numbers except odd-integer multiples of $\frac{\pi}{2}$?

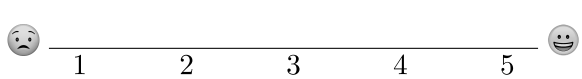
Exit 4:

Shown in Figure 6.11 is a graph of a transformation of $y = \tan(x)$. What values of C and D create this transformation?

Figure 6.11: A transformation of $y = \tan(x)$



On a scale of 1 – 5, how are you feeling with the concepts related to the graphs of other trigonometric functions? Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.



6.3 Inverse Trigonometry

In this section, we will use the inverse sine, cosine, and tangent functions, find the exact value of expressions involving the inverse-trigonometric functions, use a calculator to evaluate inverse-trigonometric functions, and find exact values of composite functions with inverse-trigonometric functions.

Textbook Reference: This relates to content in §8.3 of *Algebra and Trigonometry 2e*.

Preparation Exercises

Answer the following without using a calculator. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

Preparation 1:
Fill in the blanks referring to the angles in Figure 6.12.

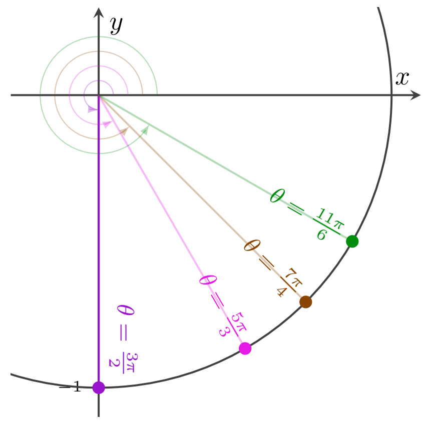
a) $\frac{11\pi}{6}$ is coterminal with the negative angle ____.

b) $\frac{7\pi}{4}$ is coterminal with the negative angle ____.

c) $\frac{5\pi}{3}$ is coterminal with the negative angle ____.

d) $\frac{3\pi}{2}$ is coterminal with the negative angle ____.

Figure 6.12: Unit Circle with Standard Values



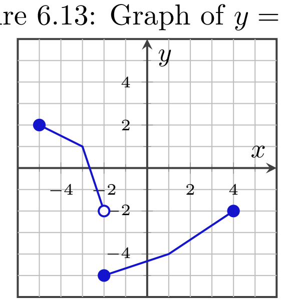
Preparation 2:
The invertible function g is defined by Table 6.1. Fill in the $g^{-1}(x)$ row. Some entries may be "undefined".

Table 6.1: The invertible function g

x	-4	-1	1	3	6	10	11
$g(x)$	10	3	6	-4	9	-1	1
$g^{-1}(x)$							

Preparation 3:
Show in Figure 6.13 is an invertible function $y = R(x)$. Draw a sketch of $y = R^{-1}(x)$ on the same figure.

Figure 6.13: Graph of $y = R(x)$



Practice Exercises

Practice 1:

Evaluate the expressions using your memorized values on the unit circle. Some might be undefined.

a) $\sin^{-1}\left(\frac{1}{2}\right)$ b) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ c) $\cos^{-1}(1)$ d) $\cos^{-1}(\pi)$ e) $\cos^{-1}\left(-\frac{1}{2}\right)$

Practice 2:

Fill in the table values using your memorized values on the unit circle.

t	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\cos^{-1}(t)$									
$\sin^{-1}(t)$									

Practice 3:

Fill in the table below with standard values of inverse trigonometric functions without a calculator. If the value of z is blank, start by filling that in first.

z	$\sin^{-1}(z)$	$\cos^{-1}(z)$	$\tan^{-1}(z)$
0			
	$-\frac{\pi}{2}$		
		$\frac{\pi}{6}$	N/A
			$\frac{\pi}{4}$
$\frac{\sqrt{2}}{2}$			N/A
		$\frac{2\pi}{3}$	N/A

Practice 4:

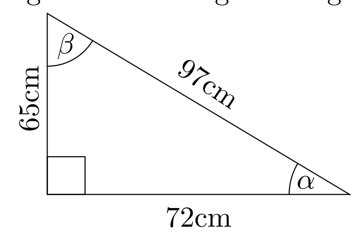
Use a calculator to approximate the values below.

a) $\sin^{-1}(0.1)$ b) $\cos^{-1}(0.6)$ c) $\tan^{-1}(2)$ d) $\sec^{-1}(-2)$ e) $\csc^{-1}(3)$

Practice 5:
Use inverse trigonometry to find the missing angles.

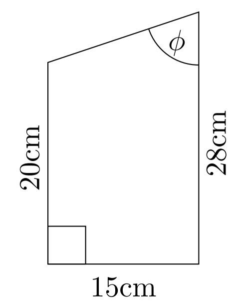
a) Use inverse trigonometry and a calculator to find the missing angles α and β in the triangle shown in Figure 6.14. Drawing not to scale.

Figure 6.14: A right triangle



b) Imagine designing a birdhouse to make out of wood. The sides of this birdhouse are shaped like trapezoids, with dimensions shown in Figure 6.15. Find the angle ϕ, in degrees, using inverse trigonometry and your calculator. Note: this angle will be related to how you set up your saw to cut the wood correctly.

Figure 6.15: The side of a birdhouse



Practice 6:
Evaluate the following expressions without a calculator.

a) $\cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right)$

b) $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$

c) $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)$

d) $\cos^{-1}\left(\cos\left(\frac{6\pi}{7}\right)\right)$

Practice 7:
Evaluate the following expressions without a calculator.

a) $\sin\left(\cos^{-1}\left(\frac{2}{3}\right)\right)$

c) $\cos\left(\sin^{-1}\left(-\frac{1}{5}\right)\right)$

b) $\sin\left(\tan^{-1}\left(\frac{4}{7}\right)\right)$

d) $\cos\left(\tan^{-1}\left(-\frac{7}{2}\right)\right)$

Practice 8:
In this exercise, discuss some of the details about inverse trigonometry with your group members.

a) What are the similarities and differences between $\sin^{-1}(x)$ and $\csc(x)$? Start by discussing the meaning of the inputs and outputs of the functions.

b) What are the similarities and differences between $\sin^{-1}(x)$ and $\sin^2(x)$? Start by discussing the meaning of the inputs and outputs of the functions.

c) The range of both $\sin^{-1}(x)$ and $\tan^{-1}(x)$ is in quadrants I and IV. The range of $\cos^{-1}(x)$ is in quadrants I and II. Why do you think that none of the inverse trigonometric functions use quadrant III?

d) If Georgiana said that, "I think that the value of $\sin^{-1}(-1)$ is $\frac{3\pi}{2}$," how would you help her understand her mistake?

Definitions

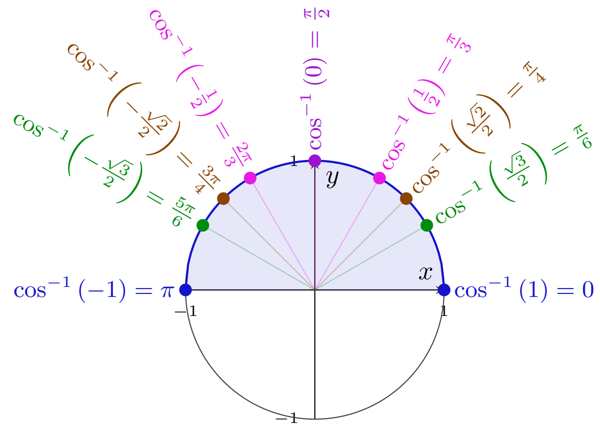
Inverse cosine

The **inverse cosine function** inputs a x-value on the unit circle and outputs the angle from 0 to π that matches that x-value, as shown in Figure 6.16. Another way to say that is that for angles, θ, in the interval $[0, \pi]$, if $\cos(\theta) = x$ then $\cos^{-1}(x) = \theta$.

Table 6.2: Domain and range for cosine and cosine inverse

Function	$\cos(z)$	$\cos^{-1}(z)$
Domain and meaning	$\mathbb{R}$ Angles on unit circle	$[-1, 1]$ x-values on unit circle
Range and meaning	$[-1, 1]$ x-values on unit circle	$[0, \pi]$ Angles on unit circle

Figure 6.16: Unit Circle with Standard Inverse Cosine Values



Inverse sine

The **inverse sine function** inputs a y-value on the unit circle and outputs the angle from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ that matches that y-value, as shown in Figure 6.17. Another way to say that is that for angles, θ, in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$, if $\sin(\theta) = y$ then $\sin^{-1}(y) = \theta$.

Figure 6.17: Unit Circle with Standard Inverse Sine Values

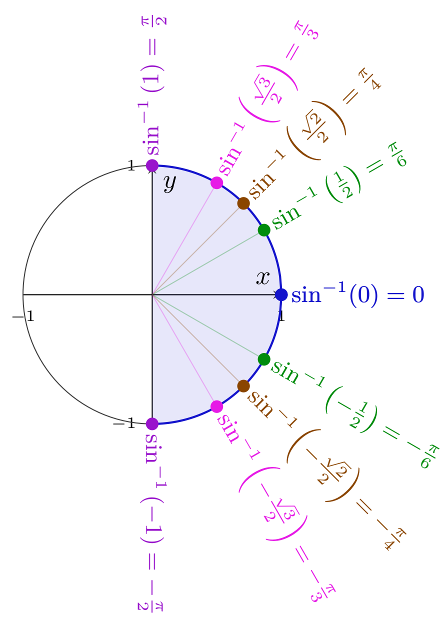


Table 6.3: Domain and range for sine and sine inverse

Function	$\sin(z)$	$\sin^{-1}(z)$
Domain and meaning	$\mathbb{R}$ Angles on unit circle	$[-1, 1]$ y-values on unit circle
Range and meaning	$[-1, 1]$ y-values on unit circle	$[-\frac{\pi}{2}, \frac{\pi}{2}]$ Angles on unit circle

Inverse tangent

The **inverse tangent function** inputs a slope, m, and outputs the angle from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ (non-inclusive) that matches that slope, as shown in Figure 6.18. Another way to say that is that for angles, θ, in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, if $\tan(\theta) = m$ then $\tan^{-1}(m) = \theta$.

Figure 6.18: Unit Circle with Standard Inverse Tangent Values

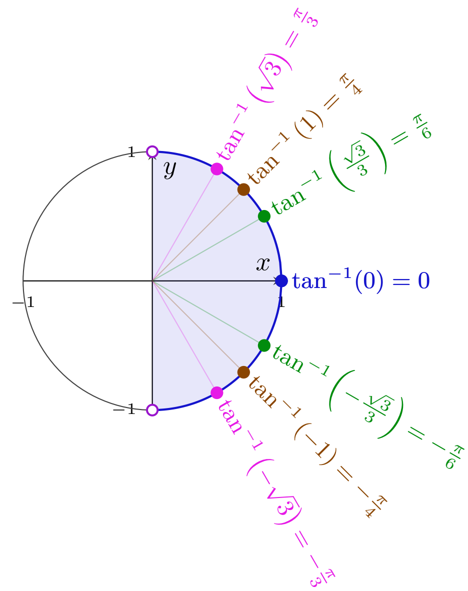


Table 6.4: Domain and range for sine and sine inverse

Function	$\tan(z)$	$\tan^{-1}(z)$
Domain and meaning	$\mathbb{R}$ except odd multiples of $\frac{\pi}{2}$ Angles on unit circle	$\mathbb{R}$ Slopes
Range and meaning	$\mathbb{R}$ Slopes	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ Angles on unit circle

Exit Exercises

Exit 1:

Fill in the blanks to show how should you think about the inputs and outputs of each function? The answer to each is either "an angle", "an x-value on the unit circle", or "a y-value on the unit circle".

a) ______________________ $= \sin\left(\text{______________________}\right)$.

b) ______________________ $= \cos\left(\text{______________________}\right)$.

c) ______________________ $= \sin^{-1}\left(\text{______________________}\right)$.

d) ______________________ $= \cos^{-1}\left(\text{______________________}\right)$.

Exit 2:

Evaluate $\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$

Exit 3:

Fill in the missing values in Table 6.5.

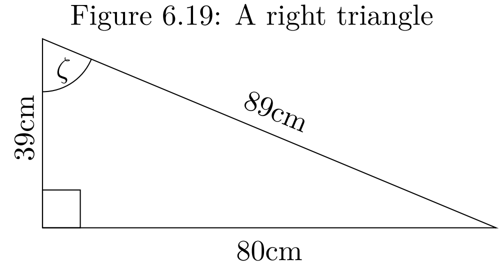
Table 6.5: A table of inverse trigonometry

z	$\sin^{-1}(z)$	$\cos^{-1}(z)$
		$\frac{\pi}{3}$
	$-\frac{\pi}{4}$	
$-\frac{\sqrt{3}}{2}$		

Exit 4:

Use inverse trigonometry and a calculator to find the missing angle ζ in the triangle shown in Figure 6.19. Drawing not to scale.

Figure 6.19: A right triangle



On a scale of 1 – 5, how are you feeling with the concepts related to inverse trigonometry?
Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.

1 2 3 4 5

Chapter 7

Trigonometric Identities

Contents

7.1 Trigonometric Identities Summary Sheet

Below is a reference sheet of all of the trigonometric identities in this chapter.

Definitions for tangent, cotangent, secant, and cosecant

- $\tan(x) = \frac{\sin(x)}{\cos(x)}$
- $\cot(x) = \frac{\cos(x)}{\sin(x)}$
- $\sec(x) = \frac{1}{\cos(x)}$
- $\csc(x) = \frac{1}{\sin(x)}$

Odd and even trigonometric functions

Sine, tangent, cotangent, and cosecant are odd functions, while cosine and secant are even functions.

- $\sin(-x) = -\sin(x)$
- $\cot(-x) = -\cot(x)$
- $\cos(-x) = \cos(x)$
- $\tan(-x) = -\tan(x)$
- $\csc(-x) = -\csc(x)$
- $\sec(-x) = \sec(x)$

Pythagorean identities

- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\tan^2(\theta) + 1 = \sec^2(\theta)$
- $\cot^2(\theta) + 1 = \csc^2(\theta)$

Sum and Difference Identities

- $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$
- $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$
- $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$
- $\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$
- $\tan(\alpha + \beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}$
- $\tan(\alpha - \beta) = \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}$

Double-angle and half angle identities

- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
  $= 1 - 2\sin^2(\theta)$
  $= 2\cos^2(\theta) - 1$
- $\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$
- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$
- $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$
- $\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}}$

Product-to-sum and sum-to-product identities

- $\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$
- $\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right]$
- $\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$
- $\cos(\alpha)\sin(\beta) = \frac{1}{2}\left[\sin(\alpha + \beta) - \sin(\alpha - \beta)\right]$
- $\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
- $\sin(\alpha) - \sin(\beta) = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$
- $\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$
- $\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

7.2 Trigonometric Identities

In this section, we will learn to verify the fundamental trigonometric identities and to simplify trigonometric expressions using algebra and the identities.

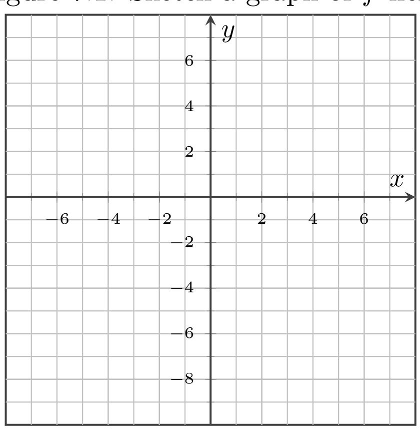
Textbook Reference: This relates to content in §9.1 of *Algebra and Trigonometry 2e*.

Preparation Exercises

Answer the following without using a calculator. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

Preparation 1:
Using a calculator, accurately sketch a graph of $f(x) = 4\sin\left(\frac{\pi}{3}(x-2)\right) + x$ on the axes shown in Figure 7.1 .

Figure 7.1: Sketch a graph of f here



Preparation 2:
If $g(x) = \frac{x-4}{x-1}$, evaluate $g\left(\frac{x-2}{x-3}\right)$ and simplify to lowest form.

Preparation 3:
Which of the six fundamental trigonometric functions are odd functions?

Preparation 4:
True or False: Since we know that $\sin^2(x) + \cos^2(x) = 1$, then if we take the square root on both sides, it must be true that $\sin(x) + \cos(x) = 1$. Explain your reasoning.

Preparation 5:
Factor the expression $1 - S^2$.

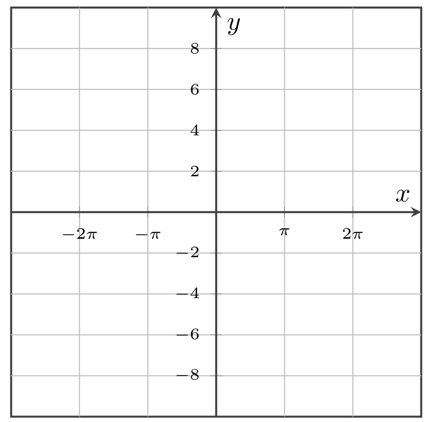
Practice Exercises

Practice 1:

Use technology to verify the identity $\sec(x) - \cos(x) = \sin(x)\tan(x)$ by following the steps below.

- Make a graph of the left-hand side of the equation onto Figure 7.2, $y = \sec(x) - \cos(x)$, using technology.
- Make a graph of the right-hand side of the equation onto Figure 7.2, $y = \sin(x)\tan(x)$, using technology.
- Describe what you see on your graph. If the two graphs above overlap, then that verifies that both sides of the equation are *identical*.

Figure 7.2: Sketch a graph here



Practice 2:

Show that $\sin(x) - \cos(x) = (\tan(x) - 1)\sec^2(x)$ is *not* an identity by creating a graph in Desmos. Discuss with your neighbor what the graph shows that makes it *not* an identity.

Practice 3:

Verify the trigonometric identities.

a) $\tan(\theta)\csc(\theta) = \sec(\theta)$

b) $\sec(-\beta) + \tan(-\beta) = (1 - \sin(\beta))\sec(\beta)$

c) $\frac{\sec(\alpha)}{\csc(\alpha)} = \tan(\alpha)$

d) $\tan(\phi)\sin(\phi) + \cos(\phi) = \sec(\phi)$

e) $\frac{\tan^2(\gamma)}{\tan^2(\gamma)+1} = \sin^2(\gamma)$

f) $\frac{\csc(\eta)\cos(\eta)}{\tan(\eta)+\cot(\eta)} = \cos^2(\eta)$

Practice 4:
Factor the trigonometric expressions.

a) $4 - 9\sin^2(x)$

b) $2\sin^2(x) - \sin(x) - 1$

Definitions

Identity

An **identity** is an equation that is always true, regardless of the value inputted.

Identities for tangent, cotangent, secant, and cosecant

Recall that the functions **tangent**, **cotangent**, **secant**, and **cosecant** can all be written in terms of sine and/or cosine.

- $\tan(x) = \frac{\sin(x)}{\cos(x)}$
- $\cot(x) = \frac{\cos(x)}{\sin(x)}$
- $\sec(x) = \frac{1}{\cos(x)}$
- $\csc(x) = \frac{1}{\sin(x)}$

Odd and even trigonometric functions

Recall that sine, tangent, cotangent, and cosecant are all **odd functions**.

- $\sin(-x) = -\sin(x)$
- $\tan(-x) = -\tan(x)$
- $\cot(-x) = -\cot(x)$
- $\csc(-x) = -\csc(x)$

Cosine and secant are **even functions**.

- $\cos(-x) = \cos(x)$
- $\sec(-x) = \sec(x)$

Pythagorean identities

Recall the three **Pythagorean Identities**.

- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\tan^2(\theta) + 1 = \sec^2(\theta)$
- $\cot^2(\theta) + 1 = \csc^2(\theta)$

Note that the first relationship is often written as either $\sin^2(\theta) = 1 - \cos^2(\theta)$ or $\cos^2(\theta) = 1 - \sin^2(\theta)$

Exit Exercises

Exit 1:
What makes an equation "an identity"?

Exit 2:
How would you decide if the equation $\sin(\theta) + \cos(\theta) = \frac{1}{\csc\theta + \sec(\theta)}$ is an identity or not?

Exit 3:
Verify the identity $-\frac{\cos(-\kappa)}{\sec(-\kappa)} = \sin^2(\kappa) - 1$

Exit 4:
Verify the identity $\frac{\cos(x)}{1-\sin(x)} = \frac{1+\sin(x)}{\cos(x)}$.

On a scale of 1 – 5, how are you feeling with the concepts related to trigonometric identities?
Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.

1 2 3 4 5

7.3 Sum and Difference Identities

In this section, we will learn to use sum and difference formulas for cosine, tangent, and to use sum and difference formulas to verify identities.

Textbook Reference: This relates to content in §9.2 of *Algebra and Trigonometry 2e*.

Preparation Exercises

Preparation 1:
Answer the following without using a calculator. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

a) Write down the memorized values from the unit circle.

i) $\sin(\pi)$

ii) $\sin\left(\frac{\pi}{3}\right)$

iii) $\sin\left(\frac{5\pi}{3}\right)$

iv) $\sin\left(\frac{3\pi}{2}\right)$

v) $\sin\left(\frac{7\pi}{6}\right)$

vi) $\sin\left(-\frac{3\pi}{4}\right)$

b) Count from 0 to π by multiples of $\frac{\pi}{12}$, and then reduce those fractions. Here's how to start out: $\frac{0\pi}{12}$, $\frac{\pi}{12}$, $\frac{2\pi}{12}$, $\frac{3\pi}{12}$, ...

c) Write down the three Pythagorean Identities.

d) True or False: $\sin(45° + 60°) = \sin(45°) + \sin(60°)$. Explain your reasoning.

e) Verify the identity $\frac{\sec(\phi)\sin(\phi)}{\tan(\phi)+\cot(\phi)} = \sin^2(\phi)$.

Practice Exercises

Practice 1:

Use the sum and difference formulas and your memorized unit circle values to evaluate the following expressions without a calculator.

a) $\sin(60° - 45°)$

b) $\sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

c) $\cos(105°)$

d) $\cos\left(\frac{17\pi}{12}\right)$

e) $\tan(75°)$

f) $\tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

Practice 2:

Imagine that there are two angles, α and β, not in the same triangle, such that $\sin(\alpha) = \frac{2}{7}$ where $\frac{\pi}{2} < \alpha < \pi$ and $\cos(\beta) = -\frac{5}{6}$ where $\pi < \beta < \frac{3\pi}{2}$.

a) Find the value of $\cos(\alpha)$.

b) Find the value of $\sin(\beta)$.

c) Find the value of $\sin(\alpha+\beta)$.

d) Find the value of $\cos(\alpha-\beta)$.

Practice 3:
Verify the identities algebraically.

a) $\cos(\lambda+\nu)\cos(\lambda-\nu)=\cos^2(\lambda)-\sin^2(\nu)$

b) $\tan\left(\rho+\frac{\pi}{4}\right)=\frac{\cos(\rho)+\sin(\rho)}{\cos(\rho)-\sin(\rho)}$

Definitions

Sum and Difference Identities

There are **sum and difference identities** for each of the trigonometric functions, but we will only focus on those for sine, cosine, and tangent.

- $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$
- $\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$
- $\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$
- $\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$
- $\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}$
- $\tan(\alpha-\beta) = \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}$

Exit Exercises

Exit 1:
Evaluate the expression $\cos(285°)$ without a calculator.

Exit 2:
Imagine that there are two angles, κ and τ, not necessarily in the same triangle, such that $\sin(\kappa) = -\frac{3}{4}$ where $\frac{3\pi}{2} < \kappa < 2\pi$ and $\cos(\tau) = \frac{3}{8}$ where $0 < \tau < \frac{\pi}{2}$.

a) Find the values of $\cos(\kappa)$ and $\sin(\tau)$.

b) Find the value of $\sin(\kappa - \tau)$.

c) Find the value of $\tan(\kappa + \tau)$.

On a scale of 1 – 5, how are you feeling with the concepts related to sum and difference identities? Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.

1 2 3 4 5

7.4 Double Angle Identities

In this section, we will learn to use double-angle and half-angle formulas to find exact trigonometric values and verify identities, and use reduction formulas to simplify an expression.

Textbook Reference: This relates to content in §9.3 of *Algebra and Trigonometry 2e*.

Preparation Exercises

Answer the following without using a calculator. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

Preparation 1:
Use the sum of angles for sine formula, $\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$, to evaluate and simplify the expression $\sin(\theta+\theta)$.

Preparation 2:
In which quadrants are the function values negative?

a) $\sin(\theta)$.

b) $\cos(\theta)$.

c) $\tan(\theta)$.

d) $\sec(\theta)$.

Preparation 3:
True or False: $\sin(60^\circ + 60^\circ) = 2\sin(60^\circ)$. Explain your reasoning.

Preparation 4:
Verify the identity $\frac{\cos(\tau)}{1+\sin(\tau)} = \sec(\tau) - \tan(\tau)$ algebraically.

Practice Exercises

Practice 1:

Find the values using the half angle identities.

a) $\sin(22.5^\circ)$

b) $\cos(15^\circ)$

Practice 2:

If $\sin(\phi) = \frac{1}{6}$ and ϕ is in quadrant II, find the following values.

a) $\cos(\phi)$

b) $\tan(\phi)$

c) $\sin(2\phi)$

d) $\cos(2\phi)$

e) $\tan(2\phi)$

f) $\sin(\frac{\phi}{2})$

g) $\cos(\frac{\phi}{2})$

Practice 3:
Verify the identities algebraically.

a) $\sec^2(\theta) = \frac{2}{1+\cos(2\theta)}$

b) $(2\sin(\theta) - 3\cos(\theta))^2 = 4 - 6\sin(2\theta) + 5\cos^2(\theta)$

Practice 4:
Using a graphing utility, find the value of C that makes the equation $(\cos(x) + \sec(x))^2 = C + \tan^2(x) - \sin^2(x)$ an identity.

Definitions

Double-Angle Identities

Each of the trigonometric functions has a formula that accounts for an angle that is twice as large as some starting angle, the **double-angle identities**. We will focus only on those for sine, cosine, and tangent. The formulas are summarized below.

- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
  $= 1 - 2\sin^2(\theta)$
  $= 2\cos^2(\theta) - 1$

- $\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$

Half-Angle Identities

Each of the trigonometric functions has a formula that accounts for an angle that is half as large as some starting angle, the **half-angle identities**. We will focus only on those for sine, cosine, and tangent. The formulas are summarized below.

- $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$

- $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$

- $\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}}$

Exit Exercises

Exit 1:
If $\tan(\rho) = \frac{4}{3}$ and ρ is in quadrant III, find the following values.

a) $\cos(\rho)$

b) $\sin(2\rho)$

c) $\sin(\frac{\rho}{2})$

Exit 2:
Algebraically verify the identity $\sin(4\alpha) = 8\sin(\alpha)\cos^3(\alpha) - 4\sin(\alpha)\cos(\alpha)$.

On a scale of 1 – 5, how are you feeling with the concepts related to double angle identities?
Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.

1 2 3 4 5

7.5 Sum-to-Product Identities

In this section, we will be made aware of the identities to express products of trigonometric functions as sums and sums of trigonometric functions as products. This is only intended to be a reference section since PCC doesn't formally cover these topics in the course. You should know that the sum-to-product and product-to-sum identities exist when you might need them.

Textbook Reference: This relates to content in §9.4 of *Algebra and Trigonometry 2e*.

Definitions

Product-to-Sum Identities

- $\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$

- $\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$

- $\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$

- $\cos(\alpha)\sin(\beta) = \frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$

Sum-to-Product Identities

- $\sin(\alpha)+\sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

- $\sin(\alpha)-\sin(\beta) = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

- $\cos(\alpha)-\cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

- $\cos(\alpha)+\cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

An example of when one of these identities might be useful is if you were trying to solve the equation $\sin(4\theta)-\sin(2\theta)=0$ by converting it into the form $\sin(\theta)\cos(3\theta)=0$. Try to finish solving this equation in Practice 5 in Section 7.6.

7.6 Solving Trigonometric Equations

In this section, we will learn to solve trigonometric equations by hand and with a calculator.

Textbook Reference: This relates to content in §9.5 of *Algebra and Trigonometry 2e*.

Preparation Exercises

Answer the following without using a calculator. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

Preparation 1:
Solve the equation $2x^2 + 5x - 6 = 0$ by hand.

Preparation 2:
Carlos and Reba were having a discussion about the equation $\sin(\theta) = \frac{1}{2}$. The answers below are either "both Reba and Carlos are right", "only Reba is right", "only Carlos is right", or "neither Reba or Carlos are right".

a) Reba said that $\frac{5\pi}{6}$ made the equation true. Carlos said that $\frac{\pi}{6}$ made the equation true. Who was right?

b) Reba said that $-\frac{5\pi}{6}$ made the equation true. Carlos said that $-\frac{\pi}{6}$ made the equation true. Who was right?

c) Reba said that $-\frac{\pi}{6}$ was the same as $\frac{11\pi}{6}$. Carlos said that $-\frac{\pi}{6}$ is *not* the same as $\frac{11\pi}{6}$. Who was right?

d) Reba said that $-\frac{7\pi}{6}$ made the equation true. Carlos said that $-\frac{11\pi}{6}$ made the equation true. Who was right?

e) Reba said that the value of $\sin^{-1}\left(\frac{1}{2}\right)$ was $\frac{5\pi}{6}$. Carlos said that the value of $\sin^{-1}\left(\frac{1}{2}\right)$ was $\frac{\pi}{6}$. Who was right?

f) Reba said that thinking about the equation $\sin(\theta) = \frac{1}{2}$ is the same as thinking about angles that have a y-value of $\frac{1}{2}$. Carlos said that thinking about the equation $\sin(\theta) = \frac{1}{2}$ is the same as thinking about angles that have a x-value of $\frac{1}{2}$. Who was right?

g) Reba said that there are two solutions to the equation $\sin(\theta) = \frac{1}{2}$. Carlos said that there are infinitely many solutions to the equation $\sin(\theta) = \frac{1}{2}$. Who was right?

Practice Exercises

Practice 1:

Practice solving the equations, on the given intervals, with your memorized unit circle values.

a) $\cos(\theta) = \frac{\sqrt{2}}{2}$ on the interval $[0, 3\pi]$.

b) $\cos(\theta) = \frac{\sqrt{2}}{2}$ on the interval $[-\pi, \pi]$.

c) $\cos(\theta) = \frac{\sqrt{2}}{2}$ on the interval $(-\infty, \infty)$.

d) $\sin(\theta) = -\frac{\sqrt{3}}{2}$ on the interval $[\pi, 4\pi]$.

e) $\sin(\theta) = -\frac{\sqrt{3}}{2}$ on the interval $[-\pi, \pi]$.

f) $\sin(\theta) = -\frac{\sqrt{3}}{2}$ on the interval $(-\infty, \infty)$.

Figure 7.3: Unit Circle with x-values of $\frac{\sqrt{2}}{2}$

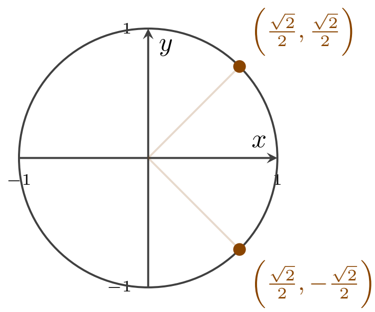
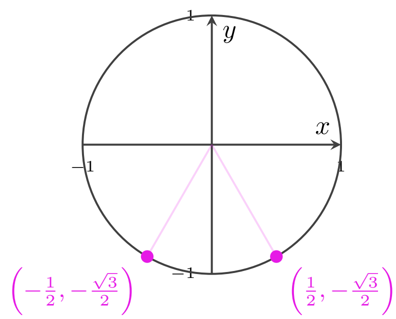


Figure 7.4: Unit Circle with y-values of $-\frac{\sqrt{3}}{2}$



Practice 2:

Solve the trigonometric equations by hand without a calculator.

a) $1 - 2\sin(\chi) = 0$ on the interval $[0, 2\pi]$.

b) $4\cos^2(\alpha) = 1$ on the interval $[-2\pi, 2\pi]$.

c) $2\cos^2(\tau) + \cos(\tau) = 1$ on the interval $[-\pi, \pi]$.

d) $\sqrt{3}\tan(\kappa) + 3 = 0$ on the interval $[0, 2\pi]$.

e) $2\sin(3\psi) = -\sqrt{3}$ on the interval $[0, \pi]$.

f) $2\cos(5\omega) + \sqrt{2} = 0$ on the interval $[0, \pi]$.

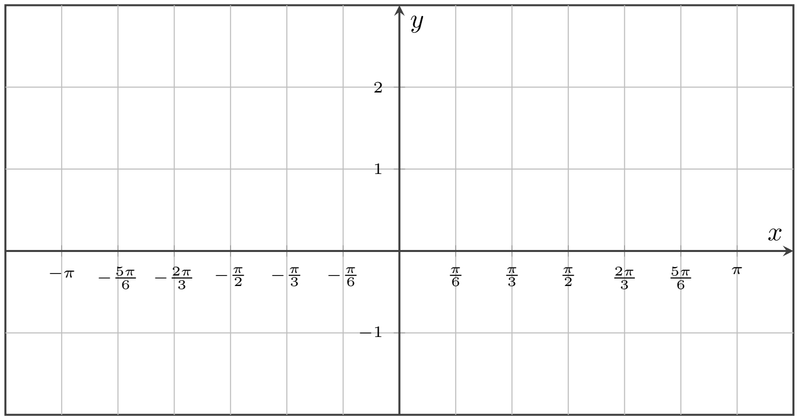
Practice 3:
Solve the trigonometric equations by hand, and then use a calculator to approximate the solutions.

a) $1 - 3\sin(\chi) = 0$ on the interval $[0, 2\pi]$.

b) $\sin^2(\phi) = 0.6$ on the interval $[0, 2\pi]$.

Practice 4:
If you were trying to solve the equation $2\sin^2(x) = \sin(x) + 1$ on the interval $[-\pi, \pi]$, and you had a graph of $y = 2\sin^2(x)$ and $y = \sin(x) + 1$, what would you look for on the graph to solve the equation? Create that graph and copy it down on the axes below, then write out the solution set based on your work.

Figure 7.5: Axes to solve the equation $2\sin^2(x) = \sin(x) + 1$ graphically



Practice 5:
Solve the trigonometric equations using identities by hand.

a) $\sin(\sigma) = -\cos(\sigma)$ on the interval $(-\infty, \infty)$.

b) $\sin(2x) = \cos(2x)$ on the interval $[-\pi, \pi]$.

c) $\sin(x) = \cos(2x)$ on the interval $[-\pi, \pi]$.

d) $\sin(4\theta) - \sin(2\theta) = 0$ on the interval $[-\pi, \pi]$ by first using the difference-to-product identity $\sin(\alpha) - \sin(\beta) = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$.

Exit Exercises

Exit 1:
Explain why you know that the equation $\cos(x) = 3$ couldn't possibly have any real solutions.

Exit 2:
Solve the equation $2\sin^2(\beta) - \sin(\beta) = 1$ on the interval $[-\pi, \pi]$ without a calculator.

Exit 3:
Solve the equation $\sin^2(\phi) = 1$ on the interval $(-\infty, \infty)$ without a calculator.

Exit 4:
Explain the solving process, in English, for the equation $\cos(\theta) = \frac{1}{2}$ on the interval $[-2\pi, 2\pi]$ step by step as if you were explaining it to someone in your class who wanted to understand today's lesson more deeply. Actually solving the equation isn't necessary.

On a scale of 1 – 5, how are you feeling with the concepts related to solving trigonometric equations? Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.

1 2 3 4 5

Chapter 8

Further Applications of Trigonometry

Contents

8.1 Laws of Sines and Cosines

In this section, we will learn how to use the laws of sines and cosines to find missing sides and angles in non-right triangles. Then we can use these laws to solve applications.

Textbook Reference: This relates to content in §10.1 and §10.2 of *Algebra and Trigonometry 2e*.

Preparation Exercises

Answer the following. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

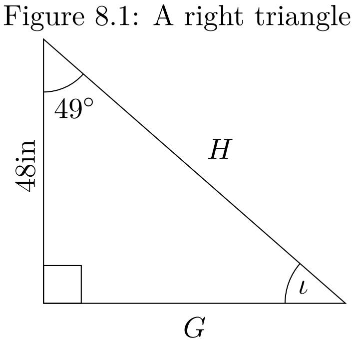
Preparation 1:
What needs to be true about a triangle in order to use the Pythagorean Theorem on it?

Preparation 2:
What needs to be true about a triangle in order to use SOHCAHTOA on it?

Preparation 3:
Find solutions to the equation $56\sin(\theta) = 21$ on the interval $[-180°, 180°]$. Use your calculator and write your answer(s) rounded to four digits behind the decimal place.

Preparation 4:
For the right triangle shown in Figure 8.1, find the missing lengths of G, H, and the angle ι. You can use a calculator to help get an approximation, accurate to 4 digits behind the decimal place.

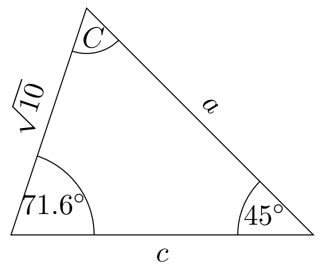
Figure 8.1: A right triangle



Practice Exercises

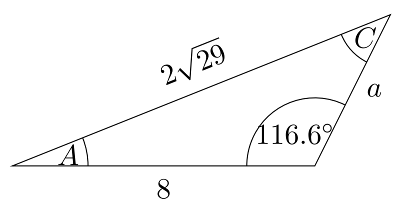
Practice 1:
Find the missing angle, C, and then use the law of sines to find the missing lengths, a and c, in the triangle in Figure 8.2. Round your answers to two digits behind the decimal place.

Figure 8.2: A triangle missing angle C and lengths a and c



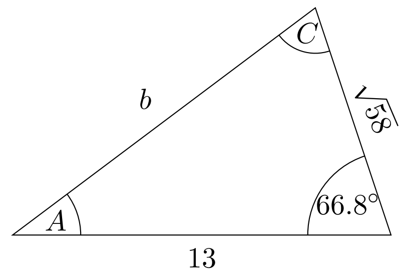
Practice 2:
Use the law of sines to find the missing angles, A and C, and the length, a, in the triangle in Figure 8.3. Round your answers to two digits behind the decimal place.

Figure 8.3: A triangle missing angles A and C, and length a



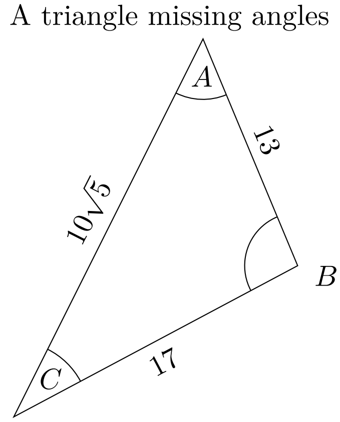
Practice 3:
Use the law of cosines to find the missing angles, A and C, and the length, b, in the triangle in Figure 8.4. Round your answers to two digits behind the decimal place.

Figure 8.4: A triangle missing angles A and C, and length b



Practice 4:
Use the law of cosines to find the missing angles, A, B, and C in the triangle in Figure 8.5. Round your answers to two digits behind the decimal place.

Figure 8.5: A triangle missing angles A, B, and C



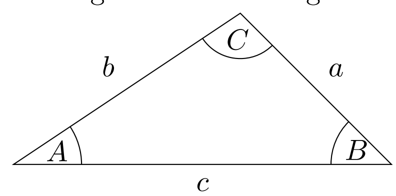
Definitions

Law of Sines

The **law of sines** says that the ratios of sines of angles in a triangle and the opposite sides are all constant. That's a bit of a mouthful, maybe a math-ful will help. Consider the triangle in Figure 8.6. The law of sines says that...

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c} \quad \text{or alternately} \quad \frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Figure 8.6: A triangle



Note: When using the law of sines, you almost never need all three parts of the equation at the same time, so you will end up using one of these at a time:

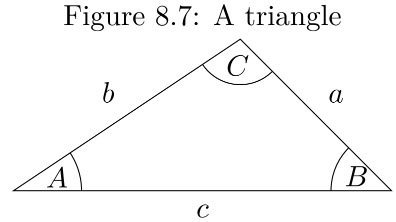
$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} \quad \text{or} \quad \frac{\sin(A)}{a} = \frac{\sin(C)}{c} \quad \text{or} \quad \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Law of Cosines

The **law of cosines** is really the natural extension of the Pythagorean Theorem for triangles without a right angle. Consider the triangle in Figure 8.7. The law of cosines says that...

$$a^2 = b^2 + c^2 - 2bc\cos(A) \quad \text{or} \quad b^2 = a^2 + c^2 - 2ac\cos(B) \quad \text{or} \quad c^2 = a^2 + b^2 - 2ab\cos(C)$$

Figure 8.7: A triangle



Note: Unlike the Pythagorean Theorem, you *don't* have to put the longest length alone on one side. It can be inputted for a, b, or c.

Exit Exercises

Exit 1:

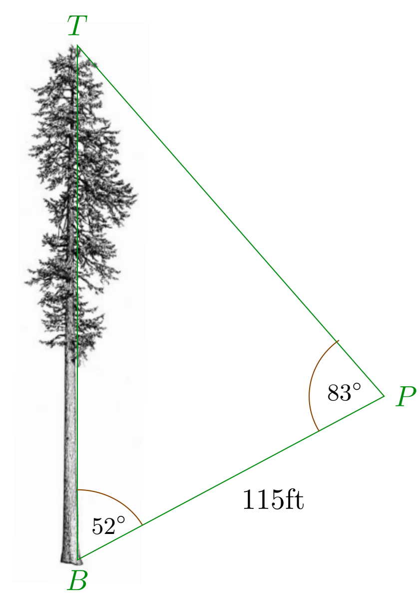
a) Michelé correctly wrote down these two equations about a triangle from a homework question.

$$b^2 = 74 + 16^2 - 2 \cdot \sqrt{74} \cdot 16 \cos(54.5°) \qquad \text{and} \qquad \frac{\sin(C)}{16} = \frac{\sin(32.5°)}{\sqrt{74}}$$

Draw the triangle from the question that these equations would have been based on.

b) On a steep hillside somewhere in the Cascades of Oregon stands a tall Douglas Fir. The slope is from point B, at the base of the tree, to the point P. The top of the tree is at point T. Some hikers wanted to get an estimate of its height, so they took a few measurements. They measured an angle of 52° from the vertically growing tree to the hillside slope using their inclinometer on their phones. Then they measured out 115ft up the hillside to point P and measured the angle from there to be 83° from the base of the tree to the top of the tree, as shown in Figure 8.8. Use the laws of sines or cosines to find an estimate for the height of the tree and round your answer to the nearest foot.

Figure 8.8: A Diagram of the Tree



c) Imagine a triangle with sides $a = 15$, $c = 8$, and the angle $C = 20°$, and length b, and angles A and B are unknown.

i) There are actually *two* such triangles that exist. Draw pictures of both triangles. *Hint: one is obtuse and the other is acute.*

ii) Write down the law of sines to solve for angle A for each triangle, and compare the two equations.

iii) Solve for the angles and missing lengths in both triangles. Round your final answers for the angles and lengths to two digits behind the decimal place. Check out this Desmos link for a visual on these "ambiguous triangles".

On a scale of 1 – 5, how are you feeling with the concepts related to the laws of sine and cosine?
Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.

1 2 3 4 5

8.2 Polar Coordinates

In this section, we will learn a completely different way of organizing the coordinate plane: polar coordinates, where a polar point, (r, θ), has a radius r from the origin and the angle from standard position, θ. We will practice plotting points using polar coordinates, converting between polar coordinates and rectangular coordinates, transforming equations between polar and rectangular forms, and graphing polar equations.

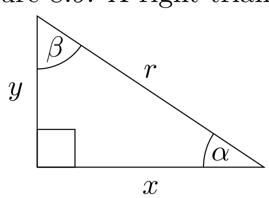
Textbook Reference: This relates to content in §10.3 and §10.4 of *Algebra and Trigonometry 2e*.

Preparation Exercises

Preparation 1:
Answer the following without using a calculator. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

Thinking back to SOHCAHTOA, recall that for a triangle with legs x and y and hypotenuse r, as shown in Figure 8.9, you know that $\cos(\theta) = \frac{x}{r}$, $\sin(\theta) = \frac{y}{r}$, and $\tan(\theta) = \frac{y}{x}$.

Figure 8.9: A right triangle



a) Solve $\sin(\theta) = \frac{y}{r}$ for y.

b) Solve $\cos(\theta) = \frac{x}{r}$ for x.

c) What would the Pythagorean Theorem say about the triangle in Figure 8.9?

Preparation 2:
What conditions would you need to put on θ, x, and y to make the solution to the equation $\tan(\theta) = \frac{y}{x}$ be $\theta = \tan^{-1}\left(\frac{y}{x}\right)$?

Preparation 3:
Which quadrant are the following angles in, when measured from the standard position.

a) $400°$ b) $-200°$ c) $-\frac{7\pi}{6}$ d) $\frac{17\pi}{3}$

Preparation 4:
Evaluate the expressions using your memorized values on the unit circle.

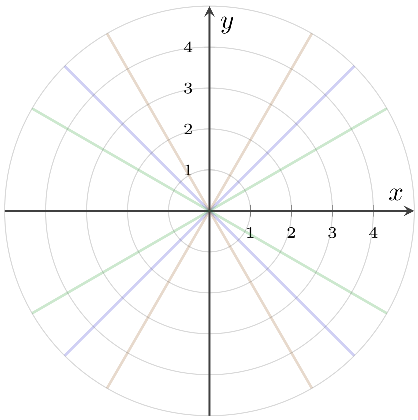
a) $\sin(420°)$ b) $\cos(-210°)$ c) $\sin\left(-\frac{7\pi}{6}\right)$ d) $\cos\left(\frac{17\pi}{3}\right)$

Practice Exercises

Practice 1:

Practice plotting points written in polar form. For clarity, a small $_p$ will be written next to polar points to distinguish them from rectangular points, which will be written with a small $_r$ next to them. Label each point.

Figure 8.10: A Polar Grid, with Standard Angles and Radii Shown



a) $\left(4, \frac{\pi}{2}\right)_p$

b) $\left(3, -\frac{\pi}{4}\right)_p$

c) $\left(-2, \frac{5\pi}{6}\right)_p$

d) $(2.5, 225°)_p$

e) $(3.5, -120°)_p$

f) $(-1.5, 270°)_p$

Practice 2:

Convert the polar points to rectangular form and the rectangular points to polar form. For clarity, a small $_p$ will be written next to polar points to distinguish them from rectangular points, which will be written with a small $_r$ next to them.

a) Find the rectangular coordinates of the polar points. Leave your answers in exact form rather than using your calculator to get a decimal value for the coordinates.

i) $\left(4, \frac{\pi}{2}\right)_p$

ii) $\left(2, -\frac{4\pi}{3}\right)_p$

b) Find the polar coordinates of the rectangular points.

i) $(1, 1)_r$. Leave your answers in exact form for both the radius and angle.

ii) $(2, 3)_r$. Leave your radius in exact form, but use a calculator to find an approximation of the angle, accurate to three digits behind the decimal place.

iii) $(-2, -3)_r$. Leave your radius in exact form, but use a calculator to find an approximation of the angle, accurate to three digits behind the decimal place.

Practice 3:

Convert equations between polar form and rectangular form. You can graph both the original equation and the polar form equation in Desmos to verify that you have converted correctly.

a) Convert the polar equations to rectangular form. You don't need to solve for a variable, necessarily.

i) $r\cos(\theta) = 5$

ii) $r = 3\sin(\theta)$

iii) $r = 5$

b) Convert the rectangular equations to polar form. You don't need to solve for a variable, necessarily.

i) $y = 3$

ii) $y = 2x + 1$

iii) $x^2 + y^2 = 2x$

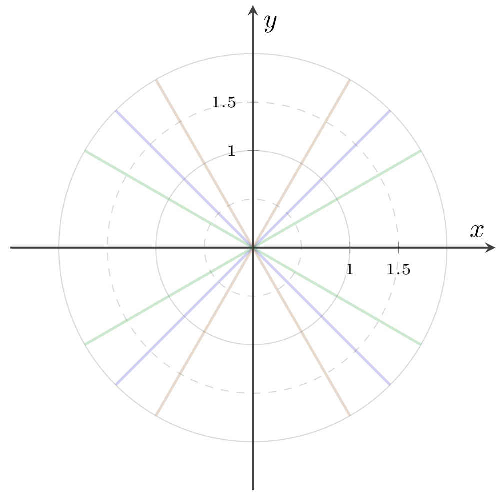
Practice 4:

Create a graph of the polar functions by filling in the tables and plotting points. Round the values of r to the nearest tenth. Note that Desmos does have polar grid in the settings, and you can graph functions of the form $r = f(\theta)$, where the θ can be found in the "A B C" keyboard. A standardized Desmos template to create this table can be found in the link.

a)

θ	$r = 2\sin(\theta)$
0°	
30°	
45°	
60°	
90°	
120°	
135°	
150°	
180°	

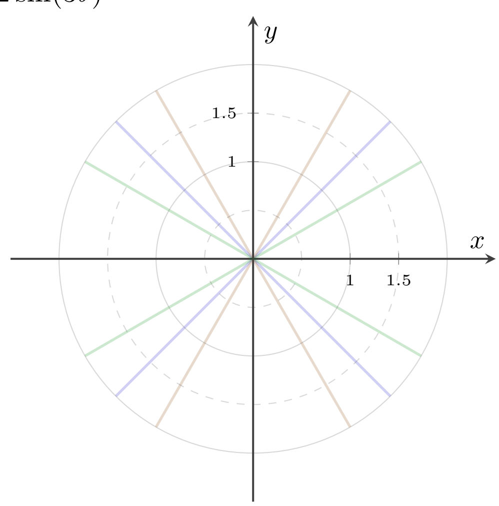
Figure 8.11: A Polar Grid for the Graph of $r = 2\sin(\theta)$



b)

θ	$r = 2\sin(3\theta)$
0°	
30°	
45°	
60°	
90°	
120°	
135°	
150°	
180°	

Figure 8.12: A Polar Grid for a Graph of $r = 2\sin(3\theta)$



c) Visit this interactive Desmos graph and change the values of a and b. Find a particular graph that you like, write down it's equation, and discuss what you like about it with your neighbor.

Definitions

Polar Functions

A **polar function** is usually given in the form $r = f(\theta)$, where θ is the angle measured from the standard position along the positive x-axis, and r is the radius measured from the origin.

Conversion formulas

- To convert from rectangular form to polar form, use these two identities.
  a) $r = \sqrt{x^2 + y^2}$ or $r^2 = x^2 + y^2$

  b) $\tan(\theta) = \frac{y}{x}$
- To convert from polar form to rectangular form, use these two identities.
  a) $x = r\cos(\theta)$

  b) $y = r\sin(\theta)$

Exit Exercises

Exit 1:
Which quadrant will the polar point $\left(-2, -\frac{7\pi}{6}\right)_p$ be in? Explain your answer.

Exit 2:
The graph of $r = 2\cos(\theta)$ is a circle of radius 1 centered at $(1, 0)_r$. Explain how r *is* a function of θ for this graph.

Exit 3:
A real-world machine used in fabrication uses polar coordinates. From a tripod setup, it has an extendable cord that comes out of a swiveling head on top of the unit that measures the distance to the end of the cord, and at what angle. This inputs a radius and angle into the computer. The machine then converts this polar point into a rectangular point and displays the resulting image on the screen. Fabrication can then begin.

a) The following polar coordinates were measured by the machine to form a new countertop. Convert them to rectangular coordinates and plot them in Desmos using the "polygon()" command. The units of measurement are in inches and degrees.

   i) $(52, 0°)_p$

   ii) $(67.158, 39.259°)_p$

   iii) $(42.5, 90°)_p$

   iv) $(18.5, 90°)_p$

   v) $(30.303, 37.626°)_p$

   vi) $(24, 0°)_p$

b) Find the total area of the countertop, in square feet. You can assume that the dimensions, as given, were intended to form right angles on the countertop.

On a scale of 1 – 5, how are you feeling with the concepts related to polar coordinates?
Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.

1 2 3 4 5

8.3 Complex Numbers

In this section, we will learn about complex numbers. We will learn how to plot complex numbers in the complex plane, convert complex numbers between polar, rectangular, and Euler forms, and how to arithmetic with complex numbers.

Textbook Reference: This relates to content in §10.5 and §2.4 of *Algebra and Trigonometry 2e*.

Preparation Exercises

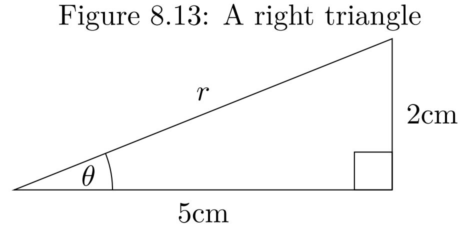
Answer the following. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

Preparation 1:
Convert the polar point $\left(4, \frac{7\pi}{6}\right)_p$ to rectangular form. Leave your components in exact form.

Preparation 2:
Convert the rectangular point $(-5, 2)_r$ to polar form. Use your calculator to find an angle in radians rounded to two digits behind the decimal place.

Preparation 3:
For the right triangle shown in Figure 8.13, find the missing length of r and the missing angle θ using SOHCAHTOA. You can use a calculator to help get an approximation, accurate to 4 digits behind the decimal place.

Figure 8.13: A right triangle



Practice Exercises

Practice 1:
Plot the complex numbers onto Figure 8.14.

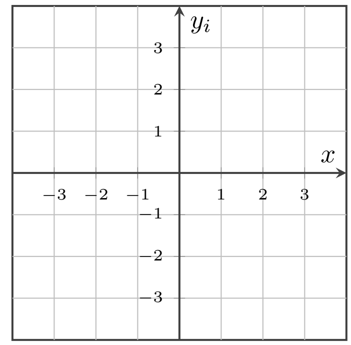
a) $2 + 3i$

b) $-1 + 2i$

c) $-3i$

d) $3 - i$

Figure 8.14: A complex plane



Practice 2:
Find the absolute value and argument of the complex numbers.

a) $2 + 3i$

b) $-1 + 2i$

Practice 3:

a) Convert the standard form complex number $5-12i$ into polar form. Use radians and round your argument to two digits behind the decimal place.

b) Convert the polar form complex number $7\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)$ into standard form. Keep your values exact using your memorized values on the unit circle.

Practice 4:
Simplify the expressions completely.

a) $(2i)^4$

b) $(2i+3)-(4i-1)$

c) $(2i+3)\cdot(4i-1)$

Practice 5:

a) Convert the standard form complex number $5-12i$ into Euler's form. Use radians and round your argument to two digits behind the decimal place.

b) Convert the Euler's form complex number $7e^{\frac{4\pi i}{3}}$ into standard form. Keep your values exact using your memorized values on the unit circle.

Definitions

The Imaginary unit i The **imaginary unit** i is defined to be $i = \sqrt{-1}$. Since there is no real number that is multiplied by itself to create a negative number, i must not be a real number. It fits into a new category of numbers called imaginary numbers which are any multiples of i. It's called a "unit" because its absolute value is 1.
For example, $4i$, $-2i$, and $\sqrt{-9}$ are all imaginary numbers.

Standard form of a Complex Number

A **complex number** is any real number plus any imaginary number. Any complex number can be written in standard form (also called rectangular form), which is the form $a + bi$, where both a and b are real numbers. For example, $3 + 0i$, $0 - 2i$, and $1 - \sqrt{5}i$ are all complex numbers.

The Complex plane

The **complex plane** is the two dimensional axes system to plot complex numbers. The horizontal axis, often just labeled x, measures the real part of a complex number. The vertical axis, often labeled y_i, measures the imaginary part of a complex number.
For example, the number $-3 + 4i$ would be plotted 3 units to the left and 4 units up from the origin at the same place that $(-3, 4)$ would have been plotted in the regular two-dimensional real axes.

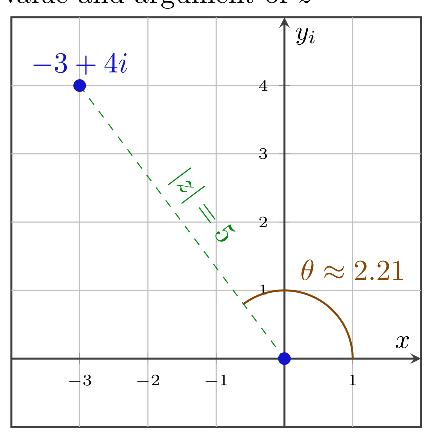
Absolute Value of a Complex number

The **absolute value of the complex number** is the distance from the origin to the number in the complex plane. The absolute value is written as usual, $|z|$. To find the absolute value of $z = a + bi$, use the Pythagorean Theorem: $|z| = \sqrt{a^2 + b^2}$.
Note: the i is *dropped* during this calculation!
Note: the absolute value of a complex number is sometimes also called the magnitude or modulus.
For Example, to find the absolute value of $z = -3 + 4i$, we would write that $|z| = \sqrt{(-3)^2 + 4^2} = 5$.

Argument of a Complex number

The **argument of the complex number**, $z = a + bi$, is an angle, θ measured from the standard position to the number in the complex plane. To find this angle, θ, use the formula $\tan(\theta) = \frac{b}{a}$ and solve for θ that is in the correct quadrant.
Note: the i is *dropped* during this calculation!
For example, to find the argument of $z = -3 + 4i$, we would first note that z is in the second quadrant, and we would write $\tan(\theta) = \frac{4}{-3}$. Next, we would take the inverse-tangent on both sides, *however*, inverse-tangent does not give us angles in quadrant II, so we know that to find the correct argument of z we should add π. So the argument of z is $\theta = \tan^{-1}\left(\frac{4}{-3}\right) + \pi \approx 2.21$, as shown in Figure 8.15.

Figure 8.15: A graph of $z = -3 + 4i$ showing the absolute value and argument of z



Polar form of a Complex number

A complex number can be written in **polar form**, which is the form $r(\cos(\theta) + i\sin(\theta))$, where $r = |z|$ and and θ is the argument of the complex number.
For example, $z = -3 + 4i$ can be written as $z \approx 5(\cos(2.21) + i\sin(2.21))$.
Note: We used the $\approx$ symbol here because the 2.21 was an approximate value.
Note: $\cos(\theta) + i\sin(\theta)$ is sometimes called $\text{cis}(\theta)$.

Euler's form of a Complex number

Euler's form of a complex number, z, is $z = re^{i\theta}$, where $r = |z|$ and θ is the argument of z.
For example, $z = -3 + 4i$ can be written as $z \approx 5e^{i \cdot 2.21}$.
Note: We used the $\approx$ symbol here because the 2.21 was an approximate value.

Exit Exercises

Exit 1:
Describe the difference between the point $(5, 8)$ in the two-dimensional real coordinate plane and the complex number $5 + 8i$ in the complex plane.

Exit 2:

a) Plot the complex number $-6 - 4i$ onto Figure 8.16.

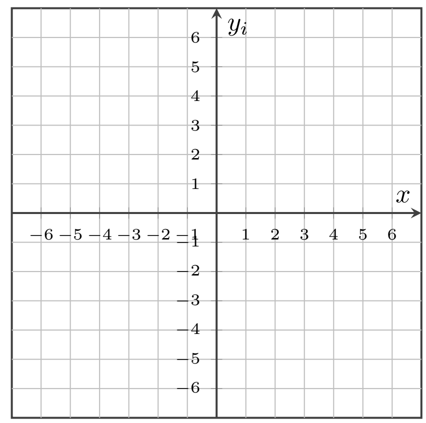
b) Find the absolute value of the complex number $-6-4i$. Simplify the radical completely and leave your answer in exact form. Illustrate this value on Figure 8.16.

c) Find the argument of the complex number $-6-4i$. Use a calculator to round your answer to three digits behind the decimal place. Illustrate this value on Figure 8.16.

d) Write the complex number $-6 - 4i$ into polar form.

e) Write the complex number $-6 - 4i$ into Euler's form.

Figure 8.16: A complex plane to plot complex numbers



On a scale of 1 – 5, how are you feeling with the concepts related to complex numbers?
Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.

1 2 3 4 5

8.4 Vectors

In this section, we will learn about vectors, which are essentially a line segment with a specific length and that points in a particular direction. We will view vectors geometrically, find the magnitude and direction of vectors, find the component form of a vector, and do arithmetic with vectors. Then we will find the dot product of two vectors and work through some applications of vectors.

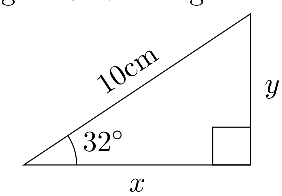
Textbook Reference: This relates to content in §10.8 of *Algebra and Trigonometry 2e*.

Preparation Exercises

Answer the following without using a calculator. These questions are intended to check the prerequisite skills needed to complete the rest of the lab.

Preparation 1:
For the right triangle shown in Figure 8.17, find the missing lengths of x and y using SOHCAHTOA. You can use a calculator to help get an approximation, accurate to 4 digits behind the decimal place.

Figure 8.17: A right triangle



Preparation 2:

a) Convert the polar point $\left(8, \frac{2\pi}{3}\right)_p$ to rectangular coordinates. Leave your coordinates in exact form using memorized values from the unit circle.

b) Write the complex number $3 + 4i$ in Euler's form, $re^{i\theta}$. Write your angle in radians and round it to 2 digits behind the decimal place.

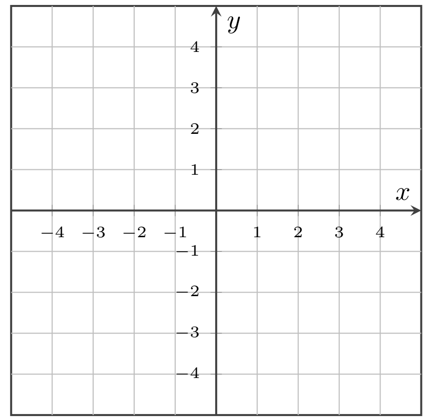
c) Convert the rectangular point $(3, 4)_r$ to polar form. Write your angle in radians and round it to 2 digits behind the decimal place.

Practice Exercises

Practice 1:

a) Draw a vector, $\vec{\mathbf{g}}$, onto Figure 8.18 that goes from point $(-3, 2)$ to point $(2, -1)$.

Figure 8.18: A grid to plot a vector



b) Draw another copy of $\vec{\mathbf{g}}$ onto Figure 8.18, this one starting from the origin.

c) Write $\vec{\mathbf{g}}$ in component form using the angled-bracket notation, $\langle \quad , \quad \rangle$.

d) Write $\vec{\mathbf{g}}$ in component form using the $a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ notation.

e) Find $\|\vec{\mathbf{g}}\|$, the magnitude of $\vec{\mathbf{g}}$.

f) Find θ, the direction of $\vec{\mathbf{g}}$, measured in degrees from the standard position. Round your answer to two digits behind the decimal place.

Practice 2:

Perform the vector arithmetic.

a) $\langle -2, 6 \rangle + \langle 4, -3 \rangle$

b) $\left(-5\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\right) - \left(6\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\right)$

c) $3\langle 4, -7 \rangle$

d) $5\left(2\hat{\mathbf{i}} - 1\hat{\mathbf{j}}\right) - 4\left(-4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)$

e) $\left(-3\hat{\mathbf{i}} + 5\hat{\mathbf{j}}\right) \cdot \left(4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\right)$

f) $\langle -6, 3 \rangle \cdot \langle 2, 9 \rangle$

Practice 3:
Are the vectors in the same direction, perpendicular, or skew? Consider using the dot product to show that vectors are perpendicular since two vectors are perpendicular if and only if their dot product is 0.

a) $\langle 6, -6\rangle$ and $\langle 1, -1\rangle$?

b) $\langle 5, -6\rangle$ and $\langle -5, 6\rangle$?

c) $\langle -6, 2\rangle$ and $\langle 3, 9\rangle$?

d) $\langle 6, -1\rangle$ and $\langle 1, -4\rangle$?

Practice 4:
Find a unit vector, $\hat{\mathbf{u}}_1$, in direction of $\vec{\mathbf{v}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$.

Practice 5:
If a vector, $\vec{\mathbf{p}}$, has a magnitude $\|\vec{\mathbf{p}}\| = 16$ and a direction angle of $\theta = 260°$, use a calculator to find the approximate components of $\vec{\mathbf{p}}$, rounded to two digits behind the decimal place.

Practice 6:
Find the angle between the two vectors $\vec{\mathbf{m}} = \langle -1, 4\rangle$ and $\vec{\mathbf{n}} = \langle 2, 1\rangle$ using the formula $\vec{\mathbf{m}} \cdot \vec{\mathbf{n}} = \|\vec{\mathbf{m}}\|\|\vec{\mathbf{n}}\| \cos(\theta)$

Practice 7:
Jamal and Jenna did a dive in the Columbia River while taking an advanced SCUBA class. It was cold, dark, and muddy. They dove to the deepest part of the river, enjoyed a fast-paced drift dive, and started toward shore well below the surface to avoid boat traffic. They swam at a pace of $2\frac{ft}{s}$ (relative to the water around them) toward shore at a bearing of 310° on his compass. The river was pushing them at a pace of $8\frac{ft}{s}$ at a bearing of 230°. They felt like they were spinning in circles, but managed to get to shore safely.

a) Find the component form of $\vec{\mathbf{s}}$, the vector that represents their swimming velocity, ignoring that the water around them is moving, assuming that the positive x-axis aligns with East.

b) Find the component form of $\vec{\mathbf{r}}$, the vector that represents the river's velocity.

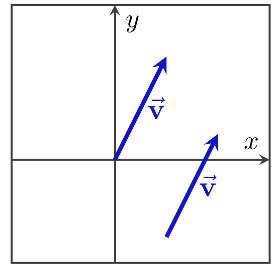
c) Find the sum $\vec{\mathbf{s}} + \vec{\mathbf{r}}$, and interpret its meaning in terms of the divers and the river.

Definitions

Vector

A **vector** is a quantity drawn as a line segment with an arrowhead at one end. It has an initial point, where it begins, and a terminal point, where it ends. It is a line segment with a direction. Vectors are placement-independent in that two vectors that are the same length and parallel are considered the same vector, as shown in Figure 8.19. Vector notation is usually either "the vector $\vec{\mathbf{v}}$" or "the vector $\vec{\mathbf{v}}$"

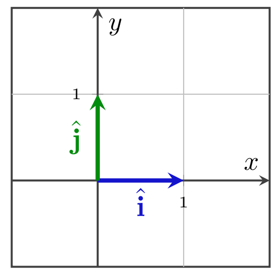
Figure 8.19: Two copies of the vector $\vec{\mathbf{v}}$



The vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$

The **vector** $\hat{\mathbf{i}}$ (read "i hat") is defined to be a vector of length 1 pointed to the right (parallel to the positive x-axis) **and** $\hat{\mathbf{j}}$ (read "j hat") is defined to be a vector of length 1 pointed up (parallel to the positive y-axis), as shown in Figure 8.20. We always use a "hat" on unit vectors instead of an arrow.

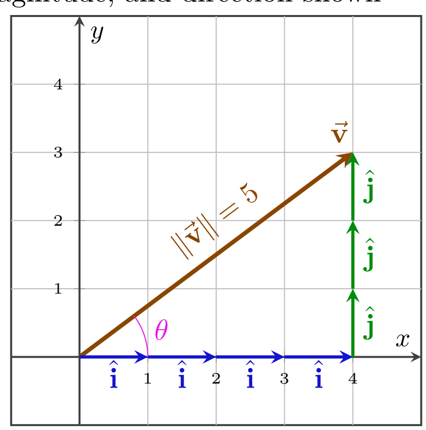
Figure 8.20: The vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$



The components of a vector

If you take a vector, $\vec{\mathbf{v}}$, and create a right triangle where the vector is the hypotenuse and the legs of the triangle are horizontal and vertical, then **the components vector** $\vec{\mathbf{v}}$ are the vertical and horizontal vectors that add together to form $\vec{\mathbf{v}}$, as shown in Figure 8.21. Since the components are horizontal and vertical, we measure them with $\hat{\mathbf{i}}$'s and $\hat{\mathbf{j}}$'s.

Figure 8.21: The vector $\vec{\mathbf{v}} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$, its components, magnitude, and direction shown



The magnitude of a vector

The **magnitude of a vector**, $\vec{\mathbf{v}} = \langle v_1, v_2 \rangle$, is the length of the vector and is written $\|\vec{\mathbf{v}}\|$ We use the Pythagorean theorem on the components of the vector to find a formula for the magnitude:

$$\|\vec{\mathbf{v}}\| = \sqrt{v_1^2 + v_2^2}$$

The direction of a vector

The **direction of a vector** is defined by an angle, θ, relative to the standard position (unless otherwise specified), positioned at the initial point of the vector.

Graphically adding two vectors

To graphically add two vectors, $\vec{\mathbf{v}} + \vec{\mathbf{w}}$, put the tail of vector $\vec{\mathbf{w}}$ at the head of $\vec{\mathbf{v}}$. The sum, $\vec{\mathbf{v}} + \vec{\mathbf{w}}$, is the vector from the tail of $\vec{\mathbf{v}}$ to the head of $\vec{\mathbf{w}}$. Check out this Desmos link for a visual on graphical vector addition.

Graphically subtracting two vectors

To graphically subtract two vectors, $\vec{\mathbf{v}} - \vec{\mathbf{w}}$, first draw the opposite vector of $\vec{\mathbf{w}}$, which would be $-\vec{\mathbf{w}}$ drawn with the same length as $\vec{\mathbf{w}}$ but exactly 180° from $\vec{\mathbf{w}}$. Then put the tail of vector $-\vec{\mathbf{w}}$ at the head of $\vec{\mathbf{v}}$. The difference, $\vec{\mathbf{v}} - \vec{\mathbf{w}}$, is the vector from the tail of $\vec{\mathbf{v}}$ to the head of $-\vec{\mathbf{w}}$. This process really creates $\vec{\mathbf{v}} + (-\vec{\mathbf{w}})$. Check out this Desmos link for a visual on graphical vector subtraction.

A unit vector

A **unit vector** is a vector of magnitude 1. There is a formula that takes any vector, $\vec{\mathbf{v}}$, and will find a unit vector, $\hat{\mathbf{u}}$, in the same direction as $\vec{\mathbf{v}}$:

$$\hat{\mathbf{u}} = \frac{\vec{\mathbf{v}}}{\|\vec{\mathbf{v}}\|}$$

The dot product of two vectors

The **dot product** of two vectors, $\vec{\mathbf{v}} = \langle v_1, v_2 \rangle$ and $\vec{\mathbf{w}} = \langle w_1, w_2 \rangle$, is the sum of the product of the corresponding components of the vectors:

$$\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = v_1 w_1 + v_2 w_2$$

This happens to tell you something about how far apart the vectors are, angle-wise. There is a relationship between the dot product and the angle, θ, between the vectors:

$$\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = \|\vec{\mathbf{v}}\| \|\vec{\mathbf{w}}\| \cos(\theta)$$

Note that this implies that two vectors are perpendicular if and only if their dot product is 0 since cosine is only 0 when the angle is a right angle. To be more specific, cosine is only 0 when the angle is an odd multiple of $\frac{\pi}{2}$.

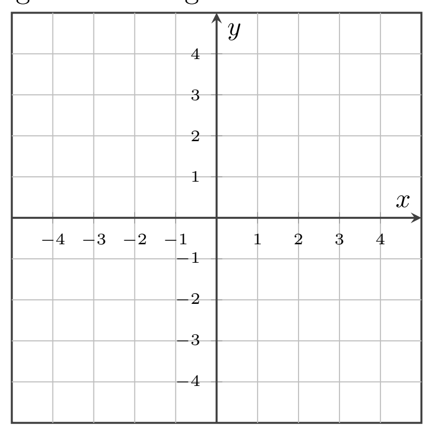
Exit Exercises

Exit 1:

Answer the following where $\vec{\mathbf{t}} = \langle -1, -3 \rangle$ and $\vec{\mathbf{s}} = \langle 3, 2 \rangle$.

a) Draw $\vec{\mathbf{t}}$ and $\vec{\mathbf{s}}$ onto Figure 8.22.

Figure 8.22: A grid to draw $\vec{\mathbf{t}}$ and $\vec{\mathbf{s}}$



b) Draw $\vec{\mathbf{t}} + \vec{\mathbf{s}}$ onto Figure 8.22.

c) Evaluate $3\vec{\mathbf{t}} - 2\vec{\mathbf{s}}$.

d) Evaluate $\vec{\mathbf{t}} \cdot \vec{\mathbf{s}}$.

e) Evaluate $\|\vec{\mathbf{t}}\|$.

f) What is the direction of $\vec{\mathbf{t}}$? Use degrees and round your answer to two digits behind the decimal place.

g) Find the angle between $\vec{\mathbf{t}}$ and $\vec{\mathbf{s}}$ using the dot product.

On a scale of 1 – 5, how are you feeling with the concepts related to vectors?
Place a mark on the scale below to indicate your overall comfort with the topics we saw in this section.

1 2 3 4 5