There are four basic types of graphs in three-dimensions. These are:

**Vector-valued functions**

A vector-valued function is a function with one input variable (called a parameter) where the output is a vector. When graphing vector-valued functions in two or three dimensions, we generally plot only the point at which the head of the vector lies when the vector’s tail is drawn at the origin. The graph of a vector-valued function is a curve.

When graphing derivative values of vector-valued functions, we tend to draw actual vectors and we place the tail of the vector at the associated point.

**Multivariable functions**

A multivariable function is a function where the input is an ordered pair and the output is a single number. In three-dimensions, the input is almost always a point in the \(xy\)-plane and the output is a value of \(z\). That is, \(z = f(x, y)\). The graph of a multivariable function in three-dimensions is called a surface.

**Implicit Equations**

An implicit equation in three-dimensions is an equation that includes at least one of the three variables \(x, y,\) and \(z\) where \(z\) has not been stated as a function of \(x\) and \(y\). Implicit equations in two variables that do not graph to planes are called cylinders in three-dimensions.

There are several implicit equations whose graphs have names. Many of these are categorized as quadric surfaces. Graphs of the basic quadric surfaces appear in section 9.6 of your text.

**Parametric Surfaces**

To generate a parametric surface, the variables \(x, y,\) and \(z\) are all stated as functions of two parameters (generally \(u\) and \(v\)).

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**Figure 1**: From left to right

- a graph of the vector-valued function \(\mathbf{r}(t) = \langle \cos(t)\sin(t), \sin(t), \cos(t)\rangle\)
- a graph of the multivariable function \(z = 0.8y\sin(x + y)\)
- a graph of the implicit equation \(z - \sin(y) = 0\)
- a graph of the parametric surface \(x = u, \ y = \sin(u)\cos(v), \ z = \sin(u)\sin(v)\)
Sketch, in one-dimension, the solutions to the equation $x = 3$.

Sketch, in two-dimensions, the solutions to the equation $x = 3$.

Sketch, in three-dimensions, the solutions to the equation $x = 3$.

Sketch, in three-dimensions, the solutions to the equation $y^2 + z^2 = 4$. 
Sketch, in three-dimensions, the solutions to the equation $y = x^2$.

Sketch, in three-dimensions, the solutions to the equation $z = 2x$. 
Consider $\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$.

What are the projections of the curve onto the coordinate planes? What are equations for the surfaces in Figure 2?
Consider \( \mathbf{r}(t) = \langle 5\sin(t), -13\cos(t), 12\sin(t) \rangle \).

What are the projections of the curve onto the coordinate planes? What are equations for the surfaces in Figure 3?
Consider \( \vec{r}(t) = (\sin(2t), \cos(2t), \cos(2t)) \).

What are the projections of the curve onto the coordinate planes?
What are equations for the surfaces in Figure 4?
Graph onto Figure 5 the vector-valued function \( \vec{r}_4(t) = \left( \frac{t\sin(t)}{\pi}, \frac{t\cos(t)}{\pi} \right) \) after first completing Table 1. Check your graph with your graphing calculator.

<table>
<thead>
<tr>
<th>Table 1: ( \vec{r}_4 )</th>
<th>Figure 5: ( \vec{r}_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( \vec{r}_4(t) )</td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
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<td>( \pi )</td>
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<td>( \frac{3\pi}{2} )</td>
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<tr>
<td>( \frac{5\pi}{2} )</td>
<td></td>
</tr>
<tr>
<td>( 3\pi )</td>
<td></td>
</tr>
</tbody>
</table>

Graph onto Figure 6 the vector-valued function \( \vec{r}_5(t) = \langle 3\cos^2(t), 2\sin(t) \rangle \) after first completing Table 2. Check your graph with your graphing calculator.

<table>
<thead>
<tr>
<th>Table 2: ( \vec{r}_5 )</th>
<th>Figure 6: ( \vec{r}_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( \vec{r}_5(t) )</td>
</tr>
<tr>
<td>0</td>
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<tr>
<td>( 3\pi )</td>
<td></td>
</tr>
</tbody>
</table>
Figures 5 - 10 on page 9 show the graphs of six different vector-valued functions along with two surfaces upon which the displayed curves lie. The formulas for three of the six functions are given below. For each function, identify the figure number whose graph corresponds to the function and identify equations for the two displayed surfaces upon which the curve lies. No other work need be shown for this problem.

The function \( \vec{r}(t) = \left( 4 \cos(t), 4 \sin(t), 4 \sin(t) + 4 \sin(2t) \right) \) is shown in Figure ________

In this figure an equation for the lighter surface is______________________________

In this figure an equation for the darker surface is______________________________

The function \( \vec{r}(t) = \left( 4 \cos(2t), 4 \sin(t), 4 \cos(2t) \right) \) is shown in Figure ________

In this figure an equation for the lighter surface is______________________________

In this figure an equation for the darker surface is______________________________

The function \( \vec{r}(t) = \left( 4 \cos(t), 4 \cos(t), 4 \sin(t) \right) \) is shown in Figure ________

In this figure an equation for the lighter surface is______________________________

In this figure an equation for the darker surface is______________________________
Graphing in three dimensions: Section 10.1

1. 

2. 

3. 

4. 

5. 

6.