1. Rigorously establish a formula for the sequence of partial sums for the series $\sum_{k=1}^{\infty} a_k$ where
\[ a_k = \frac{5}{k+3} - \frac{5}{k+5} \]. Make sure that you clearly establish that each partial sum in your work fits the formula. Then use your formula to establish the sum of the series. (12 points)
2. Rigorously establish that \( \sum_{k=1}^{\infty} \frac{5^k - 2 \cdot (-3)^{2-3k}}{2^{3k}} \) is a geometric series. Then state, with clear evidence supporting your statement, whether the series is convergent or divergent and, if the series is convergent, determine and state the sum of the series. (12 points)
3. The drug Seriesepic is prescribed for anxiety. Over a 24 hour period, a 200 lb adult male eliminates from his body 20% of the amount of Seriesepic that was in his body at the start of that 24 hour period. For example, if a 200 lb man takes a pill that contains 50 mg of the drug, 40 mg of the drug will still be in his body 24 hours after he took the pill and 32 mg of the drug will still be in his body 48 hours after he took the pill.

| 50 mg − .20(50 mg) = 40 mg and 40 mg − .20(40 mg) = 32 mg |

Suppose that the man takes a dose of \( x \) mg every 24 hours. Then in the long run the amount of drug in his body immediately \textit{before} he takes a pill is given by the sum:

\[
\text{Amount of drug still in his body from the pill he took 24 hours before.} \quad .8 x + .8^2 x + .8^3 x + .8^4 x + \cdots
\]

\[
\text{Amount of drug still in his body from the pill he took 48 hours before.}
\]

\[
\text{Amount of drug still in his body from the pill he took 72 hours before.}
\]

\[
\text{Amount of drug still in his body from the pill he took 96 hours before.}
\]

In order to remain effective, the man needs to have almost 500 mg of the drug in his body at all times. Because the amount of drug in his body builds up over time, the dosage he is given every 24 hours must be one that allows him to build up to the 500 mg level over time; i.e. he won’t reach the 500 mg level in the first few pills.

Here’s the question: How many mg of the drug should be in each pill so that \textit{in the long run} the man has almost 500 mg of the drug in his body right before he takes a pill? Make sure that you communicate how you arrive at your conclusion. You can be informal in your presentation, just make sure that your thought process is clear. (8 points)
4. Determine whether each of the following sequences is convergent or divergent and if the sequence is convergent state the value to which the sequence converges. Show work or write a few words that supports each conclusion. Again, no need to be formal, just write enough so that your thought process is clear. (4 points each)

a. The sequence generated by \( a_k = \frac{4k^{15} - 3^k}{\sqrt{25^k + k^{30}}}. \)

b. The sequence generated by \( b_k = \begin{cases} 
1 & \text{if } k = 1 \\
\frac{1}{2 - b_{k-1}} & \text{if } k > 1.
\end{cases} \)

c. The sequence of partial sums generated by the series \( \sum_{k=1}^{\infty} c_k \) where \( c_k = \begin{cases} 
5 & \text{if } k = 1 \\
\frac{4}{5}c_{k-1} & \text{if } k > 1.
\end{cases} \)
5. For each series state whether the series is convergent or divergent and state a test you could perform to establish the convergence/divergence. Do not perform the test.

A correct answer for the first series has already been supplied to help you understand the directions. (3.25 points each)

<table>
<thead>
<tr>
<th>Series</th>
<th>Con/Di-verse?</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \sum_{k=1}^{\infty} \frac{3}{2 + 5k} ]</td>
<td>Diverge</td>
<td>Limit Comparison Test with the ( p )-series [ \sum_{k=1}^{\infty} \frac{1}{k} ]</td>
</tr>
<tr>
<td>[ \sum_{k=1}^{\infty} \frac{(k-1)!}{k!} ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \sum_{k=1}^{\infty} \sin(k) ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \sum_{k=1}^{\infty} \frac{k^k}{62.8^k} ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \sum_{k=1}^{\infty} \frac{4+k^2}{5^k} ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \sum_{k=1}^{\infty} \frac{1}{\sqrt[4]{4k^3} + 1} ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \sum_{k=1}^{\infty} \sqrt[3]{\frac{3^{2k+1}}{25^{1-k}}} ]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6. Circle T for true and F for false. (1.5 points each)

   T or F  We can establish the convergence of the series \( \sum_{k=1}^{\infty} \sin^2 \left( \frac{k}{k^2} \right) \) using the integral test.

   T or F  \( \sum_{k=1}^{\infty} \cos \left( \pi k \right) \) is a geometric series.

   T or F  If \( \lim_{k \to \infty} \frac{a_k}{3^k} = 0 \), then the series \( \sum_{k=1}^{\infty} a_k \) must converge.

   T or F  If \( \lim_{k \to \infty} \frac{a_k}{3^k} = \infty \), then the series \( \sum_{k=1}^{\infty} a_k \) must diverge.

   T or F  \( \sum_{k=1}^{\infty} \frac{-2}{k} = -2 \sum_{k=1}^{\infty} \frac{1}{k} \)

   T or F  \( \sum_{k=1}^{\infty} \left( \frac{1}{k^2} + \frac{1}{k^3} \right) = \sum_{k=1}^{\infty} \frac{1}{k^2} + \sum_{k=1}^{\infty} \frac{1}{k^3} \)

   T or F  \( \sum_{k=1}^{\infty} \frac{3}{4^k} = 3 \sum_{k=1}^{\infty} \left( \frac{1}{4} \right)^k \)

7. A slope field is shown in Figure 1. Draw the solution curves for this slope field that passes through the points \((0, -4)\) and \((0, 3)\). (5 points)
8. A slope field for each of the following differential equations is shown in one of figures A - F. For each equation, state the slope field (by figure number) that matches the equation.

\[
\frac{dy}{dx} = y - x \text{ is shown in Figure } \underline{\text{A}}.
\]

\[
\frac{dy}{dx} = x + y \text{ is shown in Figure } \underline{\text{E}}.
\]

\[
\frac{dy}{dx} = 2 - y \text{ is shown in Figure } \underline{\text{C}}.
\]
9. Find the specific solution to the differential equation \( y + 2xy = y \frac{dy}{dx} + 3y^3 \frac{dy}{dx} \) where \( y(2) = -3 \). You do not need to solve your solution for \( y \). (12 points)