The Factorial Function and Simonds Theorem.

**Definition: (The Factorial Function)**
- \(0! = 1\)
- \(k! = k(k - 1)!\) for positive integer values of \(k\)

**Simonds Theorem**

For all \(p > 0\) and all \(b > 1:\)

\[
\lim_{k \to \infty} \frac{k^p}{b^k} = \lim_{k \to \infty} \frac{k^p}{k!} = \lim_{k \to \infty} \frac{b^k}{k!} = \lim_{k \to \infty} \frac{k!}{k^k} = 0
\]

**Example 1**

List, in descending order, the consecutive integer factors of each of the following.

1! =

2! =

3! =

4! =

8! =

**Example 2: Simplify each of the following.**

\[
\frac{4!}{3!} = \quad \text{Note:} \quad \frac{4!}{3!} =
\]

\[
\frac{7!}{6!} = \quad \text{Note:} \quad \frac{7!}{6!} =
\]

\[
\frac{(n + 1)!}{n!} =
\]
Example 3
Consider the sequence $a_n = (2n)!$. Simplify each of the following.

\[
\frac{a_3}{a_2} =
\]

\[
\frac{a_5}{a_4} =
\]

\[
\frac{a_{n+1}}{a_n} =
\]

Example 4
Simplify each of the indicated expressions.

Simplify $\frac{a_{k+1}}{a_k}$ for $a_k = (3k - 1)!$. 
Simplify $\frac{a_{n+1}}{a_n}$ for $a_n = 2n!$.

Evaluate $\lim_{n \to \infty} \frac{a_{n+1}}{a_n}$ for $a_n = \frac{7^n \cdot (n!)^2}{(2n-3)!}$.
Evaluate \( \lim_{k \to \infty} \frac{a_{k+1}}{a_k} \) for \( a_k = \frac{(3k)^k \cdot 2^{1-3k}}{k!} \).