Example 5
Convergent or divergent? How know? If a comparison test is needed, simply state the details of the test that needs to be applied (as well as the inevitable result, of course): do not go through the details of the test.

a. \[ \sum_{k=1}^{\infty} \frac{5 \cdot 3^k}{2^{k+1}} \]
b. \[ \sum_{k=1}^{\infty} \frac{5 + 3^k}{2^{k+1}} \]
c. \[ \sum_{k=1}^{\infty} \frac{k^k}{k!} \]
d. \[ \sum_{k=1}^{\infty} k^{-1.5} \]
e. \[ \sum_{k=1}^{\infty} \frac{1.5}{k} \]
f. \[ \sum_{k=1}^{\infty} \frac{1}{2|\cos(k)|} \]
g. \[ \sum_{k=1}^{\infty} \frac{|\cos(k)|}{k^2} \]
h. \[ \sum_{k=1}^{\infty} \frac{\ln(4)}{e^k} \]
i. \[ \sum_{k=1}^{\infty} \frac{\ln(k)}{e^k} \]

a. converge: geometric series, \( r = \frac{3}{4} \)
b. converge: LCT with geometric, \( \sum_{k=1}^{\infty} \left( \frac{3}{4} \right)^k \)
c. diverge: divergence test, \( \lim_{k \to \infty} \frac{k^k}{k!} = \infty \)
d. converge: p-series, \( \sum_{k=1}^{\infty} \frac{1}{k^{1.5}} \) (p = 1.5 > 1)
e. diverge: LCT with p-series, \( \sum_{k=1}^{\infty} \frac{k}{k^2} \) (p = 1 ≤ 1)
f. diverge: divergence test, \( \lim_{k \to \infty} \frac{1}{2\sin(k)} \) DNE.
g. converge: DCT with \( \sum_{k=1}^{\infty} \frac{1}{k} \), LCT with \( \sum_{k=1}^{\infty} \frac{1}{k^{0.4}} \)
h. converge: geometric test, \( r = \frac{1}{3} \) \( \frac{1}{3} \)
i. converge: LCT with \( \sum_{k=1}^{\infty} \left( \frac{1}{\sqrt{2}} \right)^k \)