MTH 253 Graded HW 1

This assignment is due at 6:00 PM on Tuesday, April 5

All work in this class is evaluated for presentation, notation, process, and mathematical correctness. Examples in class always follow formats that will earn you full credit provided that your mathematics is totally correct. If you have taken complete and accurate notes, then your notes serve as a guidebook for how your work should be presented and for what amount of work you are expected to show.

You are encouraged to work on these problems with your classmates. Please note, however, that copying somebody else’s solutions shortly before turning in the assignment does not constitute “working together.” What it constitutes is your getting credit for somebody else’s work. In general, if you are just copying someone else’s answers you need to come by my office or go to the tutor center to get some help; if you understood the material, you wouldn’t simply copy somebody else’s answers. Please note: working on the assignment in the classroom ten minutes or sooner before it is due is not an appropriate thing to do.

1. Find the solution to the differential equation \( xy \frac{dy}{dx} = 1 + y^2 \) that lies entirely in Quadrant I and passes through the point \((1,2)\). State your final solution in the form \( y = f(x) \) (i.e., isolate \( y \)).

Graph the solution curve onto Figure 1.

\[
\begin{align*}
    xy \frac{dy}{dx} &= 1 + y^2 \\
    \frac{y}{1+y^2} &= \frac{1}{x} \frac{dx}{dx} \\
    \int \frac{y}{1+y^2} \, dy &= \int \frac{1}{x} \, dx \\
    \frac{1}{2} \ln (1+y^2) &= \frac{1}{2} \ln (1x) + C
\end{align*}
\]

\[
\begin{align*}
    \ln (1+y^2) &= 2 \ln (1x) + C_i \\
    1+y^2 &= e^{2\ln (1x)} + C_i \\
    1+y^2 &= e^{\ln (x^2)} e^{C_i} \\
    y^2 &= C_2 x^2 - 1 \text{ (general solution)} \\
    y(1) &= 2 \Rightarrow 2^2 &= C_2 (1^2) - 1 \Rightarrow C_2 = 5 \\
    \text{The specific solution is} \\
    y &= \sqrt{5x^2 - 1} \text{ (in quadrant I)}
\end{align*}
\]
2. Detective Toad came upon the dead body of Freddie Frog. Detective Toad cursed the amphibious nature of frogs, because the variable body temperature of a live frog meant that the temperature of Freddie’s body upon discovery was of no use in determining the time of Freddie’s death. Never-the-less, Freddie’s body certainly held clues as to the manner of death, so Detective Toad called for a bus to haul the frog off to the cold storage unit in the medical examiner’s office. The body temperature of the frog when it went into cold storage was 82°F and two hours later the body temp was 68°F. The cold storage unit was kept at a temperature of 38°F. How much time elapsed after being placed into the cold storage unit before the frog’s body temperature was within 10% of the temperature inside the cold storage unit?

Use $T$ as the temperature (°F) $t$ hours after the frog’s body was placed into cold storage.

\[
\frac{dT}{dt} = k \\
\int \frac{1}{T-38} \, dt = \int k \, dt \\
\ln|T-38| = kt + C \\
|T-38| = e^{kt+C} \\
T-38 = e^{kt+C} (T>38) \\
T = 38 + Ce^{kt} \text{ (general solution)}
\]

\[
T(0) = 82 \implies 38 + C, \quad e^{0} = 38 \\
\implies C = 44 \\
\implies T = 38 + 44e^{kt} \\
T(2) = 68 \implies 38 + 44e^{2k} = 68 \\
\implies e^{2k} = \frac{15}{22} \\
\implies e^{k} = \sqrt[2]{\frac{15}{22}} \\
\implies T = 38 + 44\left(\sqrt[2]{\frac{15}{22}}\right)^{t/2}
\]

We need to determine when the frog’s temperature reached 110% of 38°F.

\[
38 + 44\left(\sqrt[2]{\frac{15}{22}}\right)^{t/2} = 41.8 \\
\implies t \approx 12.8
\]

Once placed in storage, it took about 12.8 hours for the frog’s temperature to fall within 10% of the ambient temperature in the cooling unit.