General Directions: For each problem, your ability to label the pictures and document your work in a manner consistent with that illustrated and discussed in class will be evaluated along with your ability to find the “correct answer.” Unless otherwise stated in a specific problem, you may use your calculator to evaluate all integrals.

1. The boundaries of the region outlined in Figure 1 are the parabola \( y = 6 - x^2 \) and the bottom half of the parabola \( x = 2 - \frac{(y - 2)^2}{5} \). Find the area of the heavily outlined region. (12 points)

You may not get help of any kind related to questions on this test. Any and all questions you ask must be directed to Mr. Simonds and nobody else (including fellow students). You may refer to your notes, text, and anything posted on Mr. Simonds spot account. You may not troll the internet for solutions.
2. Use the **washer method** to find the volume of the solid that results from rotating about the line $x = 2$ the region bounded above by the curve $y = 4 - x^2$ and below by $y = 1$. (12 points)

![Figure 2: Formulas Diagram](image)
3. Use the shell method to find the volume of the solid that results from rotating about the line \( x = 2 \) the region bounded above by the curve \( y = 4 - x^2 \) and below by \( y = 1 \). (12 points)
4. A certain spring has a natural length is 66 inches. A force of 1.7 lb is required to hold the spring at a length of 72 in. How much work is done while stretching the spring from its natural length to a length of 72 in? **You may not use your calculator while working this problem. You need to integrate by hand and you need to include appropriate units in all calculations.** (12 points)

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5. Suppose that $T(t)$ is the temperature (°F) at the center of an uneaten hunk of cheese $t$ hours after the cheese is taken out of the refrigerator and placed onto the kitchen island (where it sits for the next four hours). Suppose further that $T'(t) = 36e^{-1.2t}$.

   a. Find the average value of $T'(t)$ over $[0,3]$ and explain the contextual meaning of that value. Round the value to the nearest hundredth. Make sure that your conclusion includes relevant units. (8 points)

   b. Suppose that three hours after the cheese is placed on the kitchen counter the temperature at the center of the cheese is $72^\circ$F. What, to the nearest hundredth degree, was the temperature at the center of the cheese the instant it was removed from the fridge? Make sure that both your reasoning and your conclusion are clear (and include relevant units). (8 points)
6. Evaluate – completely without the use of your calculator - \[ \int_{1}^{10} \frac{x}{\sqrt{x-1}} \, dx \]. (12 points)
7. Evaluate – completely without the use of your calculator - \( \int_{e}^{\infty} \frac{1}{x \ln(x)} \, dx \). (12 points)

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8. Answer each question on this page in reference to the function \( f \) shown in Figure 5; you may simply supply the requested values in the provided blanks – no explanations necessary. The “areas” of the three shaded regions are, from left to right, 4, 8, and 3.4. Assume that \( F \) is in reference to the specific antiderivative of \( f \) that passes through the point \((3, -8)\); i.e., \( F(3) = -8 \).

(12 points total)

a. \( F(5) = \) __________

b. \( F(0) = \) __________

c. \( \int_{3}^{5} f(x) \, dx = \) __________

d. \( \int_{0}^{-2} f(x) \, dx = \) __________

e. \( \int_{-1}^{0} f(2x) \, dx = \) __________

f. If \( g(t) = \int_{5}^{t} f(x) \, dx \), then \( g(0) = \) __________

g. Circle each of the following that must have a negative value.

\[
\begin{align*}
\int_{-2}^{5} f(x) \, dx & \quad \int_{-2}^{5} F(x) \, dx & \quad \int_{-2}^{5} f''(x) \, dx \\
\int_{5}^{-2} f(x) \, dx & \quad \int_{5}^{-2} F(x) \, dx & \quad \int_{5}^{-2} f'(x) \, dx
\end{align*}
\]