MTH 252 – Winter Term 2015
Test 1 - Memory Portion
Given: January 27, 2015

You must turn in this portion of the test to retrieve the remainder of the test (and the formula sheet). You **may not** come back to this portion of the test ... once it is turned in it is in.

There are questions on both sides of this portion of this paper.

1. State the form of each limit in its provided blank. That’s it – there’s nothing more to this question.
   **Note:** Do not change the limit expression in any manner what-so-ever. (6 points)

   The form of \( \lim_{x \to 0^+} x^{\ln(x)} \) is __________. The form of \( \lim_{x \to \infty} \left( \ln(x) - \frac{2 - x}{5} \right) \) is __________.

   The form of \( \lim_{x \to \frac{\pi}{2}^-} \tan \left( \frac{x}{2} \right) \) is __________. The form of \( \lim_{x \to \infty} \left( \sqrt{e} - \tan^{-1}(x) \right) \) is __________.

2. Complete each blank with the correct option, the two options being “is” or “is not.” (6 points)

   The limit form \( 0^\infty \) ________________ indeterminate.

   The limit form \( 0^0 \) ________________ indeterminate.

   The limit form \( \infty^0 \) ________________ indeterminate.

   The limit form \( \infty^\infty \) ________________ indeterminate.

   The limit form \( \frac{\infty}{0} \) ________________ indeterminate.

   The limit form \( 0 \cdot \infty \) ________________ indeterminate.
3. Suppose that $f$ is a function that is continuous everywhere. Suppose further that $f'(8) = 0$, $f''(8) = 0$, and $f'''(8) < 0$. (This entire question worth 9 points total)

**Part 1:** Complete each sentence by writing the appropriate word(s) into the provided blank(s).

*Word choices:* positive, negative, increasing, decreasing, concave up, concave down

a. Since $f'''(8) < 0$, $f''$ must be __________ at 8.

b. Together with fact (a), since $f''(8) = 0$,

the value of $f''$ must change from __________ to __________ at 8.

c. Given fact (b),

$f$ must change from __________ to __________ at 8.

**Part 2:** Given fact (c), draw on an axis system a sketch of what $f$ looks like in the vicinity of $x = 8$ (assume that $f(8) = 5$). Given your sketch, what sort of point must $f$ have at 8?
1. Use the Absolute Max/Min Theorem to determine the absolute minimum value of the function
   \( f(x) = x^3 - 6x^2 + 12x - 3 \) over the interval \([0, 4]\). Make sure that you show work consistent with that illustrated and discussed during class and make sure that your conclusion is clear and on point. (9 points)
2. Write each general antiderivative into its provide blank. No other work need be nor should be shown – do any necessary “figuring” on your scratch paper. (14 points total)

\[ \int \frac{2}{x-3} \, dx = \quad \int e^{y/3} \, dt = \]

\[ \int \frac{8x^2 - 7x}{x} \, dx = \quad \int \frac{1}{2\sqrt{x+4}} \, dx = \]

3. The acceleration on Mars due to the planet’s gravitational force is constant and roughly 12 ft/s/s. Some day on Mars, Ester Naught will throw a ball straight upward. At the instant Ms. Naught releases the ball it will be 4 feet above the surface of Mars and will be traveling at a velocity of 36 ft/sec. What will be the maximum height reached by the ball? Make sure that you show all relevant work and that the purpose of each calculation is clear. Make sure that you state a clear conclusion. (15 points)
4. Evaluate each limit showing all relevant steps. To earn full credit all work must be shown in a manner consistent with that discussed and illustrated in class. This problem continues on page 4.

a. Evaluate \( \lim_{x \to \infty} \left( 1 + \frac{2}{x} \right)^x \). (10 points)
b. Evaluate \( \lim_{x \to 0^+} \left( \frac{2}{\sin(2x)} - \frac{1}{x} \right) \). (10 points)
On this portion of the test you are expected to take all derivatives and perform all nontrivial algebra and arithmetic using your calculator.

Don’t forget that all derivative formulas need to be completely simplified; this includes, but is not limited to, finding the completely factored form of the derivative formula.

To earn full credit, your solutions need to include all relevant information and exclude any irrelevant information. You should use appropriate calculus based techniques as illustrated in class.

1. Consider the function \( f(x) = \frac{\sqrt[3]{(x+1)^3}}{(x-1)^3} \). Find the critical numbers of \( f \), showing all of the details that go into your determination. Make sure that you present your work in a manner that is consistent with that demonstrated and discussed in class. (9 points)
2. Find the stationary numbers of the function \( f(x) = e^{-x} \cdot x^{\frac{3}{4}} \). Then perform a second derivative test at each stationary number and state the appropriate conclusions. Make sure that you present your work in a manner that is consistent with that demonstrated and discussed in class. (12 points)