MTH 252 – Practice Test 1 – Version B

On this portion of the test (the first three questions) it is expected that you will use your calculator to take all derivatives and to perform all non-trivial algebra. When taking derivatives you will not receive full credit unless the stated form of the derivative is completely simplified as spelled out and exemplified in your lecture notes.

For each problem, to earn full credit your work must be outlined in a manner consistent with that discussed and exemplified during lecture.

1. Find the critical numbers of the function \[ g(t) = \frac{(t - 4)^{2/3}}{t + 4}. \]

2. Find the stationary numbers for the function \[ f(x) = (x - 2)^2 \sqrt{x + 3}. \] Then perform a second derivative test at each stationary number and state appropriate conclusions based upon the results of the tests.

3. State all local minimum and local maximum points on \[ q(x) = \frac{(x - 8)^{2/3}}{(x - 3)^4} \] after first establishing the critical numbers of \( q \) and creating a well-documented increasing/decreasing table for \( q \).

You may not use your calculator while working the following problems.

For each problem, to earn full credit your work must be outlined in a manner consistent with that discussed and exemplified during lecture.

1. Find the absolute maximum value of the function \( g(x) = (x - 2)^2 (x - 5) \) over the interval \([3, 6]\). Make sure that your reasoning is clear.

2. State all values of \( x \) where the curve \( y = x^2 \ln(x - 2) \) crosses the \( x \)-axis. Write a brief explanation.

3. For a certain function, \( z \), the first derivative function is shown in Figure 1. Determine the values of \( t \) over the interval \([0, 2\pi]\) where \( z \) has its absolute extreme values. Make sure that your reasoning is clear.

Please note that \( z' \) is everywhere continuous so it follows that \( z \) is also everywhere continuous. You may skip addressing the continuity issues on this problem.

![Figure 1: \( y = z'(t) \)](image)
4. For each limit in Table 1, state the form of the limit and whether or not the limit is of indeterminate form. If the limit is not of indeterminate form, state its value (or $\infty$ or $-\infty$) in the provided space in the table. If the limit is of indeterminate form, find its value on the provided blank sheet of paper using methods and organizational strategies consistent with those discussed and illustrated during class. (The test this question came from had a blank sheet of paper attached to the test.)

Table 1: Limit questions and answers

<table>
<thead>
<tr>
<th>Limit</th>
<th>Form</th>
<th>Indeterminate? (yes or no)</th>
<th>Value (or $\infty$ or $-\infty$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lim_{x \to 0} \frac{\sin^2(x)}{4x^2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lim_{\alpha \to \pi} \frac{\cos(\alpha) - 1}{\cos(\alpha) + 1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lim_{y \to 0} [\cot(y) - \sec(y)]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lim_{t \to 0} [t \ln(t)]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lim_{x \to \infty} \frac{e^{-x}}{x^3}$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\lim_{x \to \infty} \frac{\tan^{-1}(x)}{\tan\left(\frac{1}{x}\right)}$</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Find each antiderivative.

a. Find $\int \left(10\sqrt{x} - 4\cos(x)\right) \, dx$

b. Find $\int (t + 3)^2 \, dt$

c. Find $\int e^x \, d\theta$

d. Find $\int \frac{\sqrt{x^5} - 6}{x} \, dx$

e. Find $\int (\sin^2(x) + \cos^2(x)) \, dx$

f. Find the antiderivative of $f(x) = \sec^2(x) - e^{2x}$ with the property that $F(0) = 4$. 