MTH 252 Practice Test 1 – Version A

All work on this test will be evaluated for your presentation as well as for the "correctness" of your "answer."

**Calculator Questions**

1. Find the stationary numbers of the function \( g(t) = \frac{\sqrt{t - 3}}{t - 10} \) after first finding, and completely simplifying, the formula for \( g'(t) \). (Use your calculator to find and simplify the formula for \( g'(t) \).) Then perform a second derivative test at each stationary number and state appropriate conclusions. Show all relevant work in a well-organized and well outlined manner.

2. Find the critical numbers of the function \( f(x) = \frac{(x + 2)^2}{(x - 3)^{1/3}} \), build an increasing/decreasing table for \( f' \), and then state all local minimum and maximum points on \( f \). Use your calculator to find, and completely simplify, the formula for \( f'(x) \). Show all relevant work in a well-organized and well outlined manner.

3. If 1200 cm\(^2\) of material is available to make a box with a square base and an open top, find the largest possible volume of the box. Show all relevant work in a well-organized and well outlined manner.

**No Calculator Questions**

1. Find the absolute maximum value of the function \( f(x) = -5 + 72x - 15x^2 + x^3 \) over the interval \([0, 2]\). Show all relevant work in an organized and clearly outlined manner.

2. Complete each limit equation with the correct value or symbol.

   a. \( \lim_{t \to \infty} \tan^{-1}(t) = \)
   
   b. \( \lim_{t \to \infty} \sec(t) = \)

   c. \( \lim_{\theta \to 0} \tan(\theta) = \)
   
   d. \( \lim_{x \to -\infty} e^x = \)

3. Evaluate each limit using L’Hopital’s Rule where appropriate. Show all relevant steps as illustrated repeatedly in class. Don’t forget to write your own subject!

   a. \( \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin(x)}{(2x - \pi)^2} \)
   
   b. \( \lim_{t \to -\infty} \ln(1 + 2e^t) \)

   c. \( \lim_{x \to \infty} \left[ xe^{1/x} - x \right] \)
   
   d. \( \lim_{x \to 0^+} \sin(x)^{\tan(x)} \)
4. In each of parts (a), (b), and (c), a function and its first derivative are given. For each pair, state any and all critical numbers of the function in the provided blank. No other work need be, nor should be, shown.

a. \( h(x) = (x - 7)(x + 3) \) and \( h'(x) = 2x - 4 \).

The critical number(s) of \( h \) are (is) _________________________________.

b. \( g(x) = \frac{x - 3}{(6 - x)^3} \) and \( g'(x) = \frac{x}{(6 - x)^3} \).

The critical number(s) of \( g \) are (is) _________________________________.

c. \( f(x) = \frac{\sqrt{x - 10}}{x - 20} \) and \( f'(x) = \frac{-x}{2(x - 20)^2 \sqrt{x - 10}} \).

The critical number(s) of \( f \) are (is) _________________________________.

5. State, in the provided spaces in Table 1, the form of the given limit and whether or not the limit is of indeterminate form.

<table>
<thead>
<tr>
<th>Limit</th>
<th>Form of limit</th>
<th>Is limit of indeterminate form?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lim_{t \to \infty} \frac{\sin \left( \frac{1}{t} \right)}{\ln \left( \frac{1}{t} \right)} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lim_{x \to 0^+} (\sqrt{x \ln(x)}) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lim_{x \to 0^+} \left[ (1 - 2x)^{\frac{1}{x}} \right] )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lim_{x \to 0^+} \left[ (1 - 2x)^{\frac{1}{x}} \right] )</td>
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</tbody>
</table>
6. For the function \( g(x) = (x-3)^4 \), the first two derivatives are \( g'(x) = 4(x-3)^3 \) and \( g''(x) = 12(x-3)^2 \). What sort of point does \( g \) have at \( x = 3 \): a local maximum point, a local minimum point, an inflection point, or none of these? Justify your answer.

7. Find each antiderivative.
   
   a. \( \int 4 \csc^2(t) \, dt \)  
   b. \( \int (3e^x - 6\sqrt{x}) \, dx \)  
   c. \( \int (4x^2 - 2)^2 \, dx \)  
   
   d. \( \int (\sin(\theta) - \cos(\theta)) \, d\theta \)  
   e. \( \int \frac{t^6 - \sqrt{t}}{\sqrt[3]{t}} \, dt \)  
   f. \( \int e^{7x} \, dx \) (Hint: Write down a guess, check, and then fix your answer as necessary.)

8. Find \( y(t) \) if \( y''(t) = -\frac{3}{t^2} \), \( y'(1) = 6 \) and \( y(1) = 4 \).

9. A car accelerates from a stop at a constant rate and after 10 seconds has travelled 400 ft. What is the constant rate of acceleration?

10. State whether or not each given scenario is possible, and give a brief explanation for your decision. An accurate and well labeled picture could serve as an adequate explanation. (A picture is worth a thousand words, eh?)
   
   a. Is it possible for a function to have 2 local maximum points but no local minimum point?
   
   b. Is it possible for a function to have a local maximum point but no absolute maximum value over \((-\infty, \infty)\)?