Key Concepts:  Areas of regions in a plane

Suppose that the functions \( y = T(x) \) and \( y = B(x) \) are both continuous over \([x_L, x_R]\) and that \( T(x) \geq B(x) \) over that entire interval. Then the area of the region bounded by \( T \) and \( B \) over \([x_L, x_R]\) is given by \( \int_{x_L}^{x_R} h(x) \, dx \) where \( h(x) = T(x) - B(x) \).

Suppose that the functions \( x = R(y) \) and \( x = L(y) \) are both continuous over \([y_a, y_T]\) (on the \( y \)-axis) and that \( R(y) \geq L(y) \) over that entire interval. Then the area of the region bounded by \( R \) and \( L \) over \([y_a, y_T]\) is given by \( \int_{y_a}^{y_T} h(y) \, dy \) where \( h(y) = R(y) - L(y) \).

Find the area of the region bounded by the parabolas \( y = -\frac{3}{4}x^2 + \frac{1}{4}x + 5 \) and \( y = \frac{1}{8}x^2 - \frac{5}{4}x - 3 \).

Please note that I am demonstrating the work I actually expect to see on your paper when I ask a similar problem on a test or a graded hw assignment.
Find the area of the region bounded by the curves $x = 2y - 4$ and $x = y^2 - 5y + 2$. Please note that I am demonstrating the work I actually expect to see on your paper when I ask a similar problem on a test or a graded hw assignment.
Once again find the area of the region bounded by the curves $x = 2y - 4$ and $x = y^2 - 5y + 2$, only this time do so after making the entirely silly decision that you must work the problem using the top/bottom boundaries. Please note that I am demonstrating the work I actually expect to see on your paper when I ask a similar problem on a test or a graded hw assignment.
Find the formula for the average value of the function $y = f(x)$ over the interval $[a, b]$. 
The Mean Value Theorem for Integrals says that if \( f \) is continuous over \([a, b]\), then there must be at least one value, \( c \), on \([a, b]\) where

\[
f(c) = \frac{1}{b - a} \int_a^b f(x) \, dx.
\]

Find said values for the function \( f(x) = 3x^2 - 12x + 18 \) over the interval \([2, 5]\).

By definition, the work done when constant force, \( F \), is applied over a distance, \( d \), is \( W = F \, d \). Find the formula for the amount of work done over the interval \([a, b]\) if a variable force, \( F(x) \), is applied over this interval.
Find the work done when you lift a 50 lb bag a total of 5 feet.

A certain spring has a natural length of 6 inches. When the spring hangs vertically and a 10 lb weight is added to the end of the spring, the spring comes to rest at a length of a foot and a quarter. How much work is required to stretch the spring from 6 inches to a foot and a quarter.

Hooke’s Law
The force required to hold a spring $x$ units beyond its natural length is given by the formula $F(x) = kx$ where $k$ is called the spring constant.
Cowtown is in a slump. \( I(t) \) represents the rate at which new residents settled into Cowtown (via birth or migration) \( t \) days after 12:00:01 am, January 1, 2007. \( O(t) \) represents the rate at which residents left Cowtown for good (via death or migration) \( t \) days after 12:00:01 am, January 1, 2007. Both \( I(t) \) and \( O(t) \) are measured in people/day.

a. Suppose that \( I(t) = 4 + 3\cos\left(\frac{\pi}{2}t\right) \) and \( O(t) = 18 - \sin\left(\frac{\pi}{2}t\right) \). What, including unit, is the average value of the function \( f(t) = I(t) - O(t) \) over the interval \([182.5, 365]\). Show the integral used in your calculation.

b. State the practical meaning of the value found in part a. i.e., what does the value tell you about the population of Cowtown?

c. If Cowtown had 100,000 residents at 12:00:01 pm, July 2, 2007, how many residents did it have at the end of 2007? Make sure that you explicitly show how you arrived at your conclusion.
It requires 7 J to stretch a certain spring from a length of 8 m to a length of 9 m and another 14 J to stretch the spring from 9 m to 10 m. Find the spring constant and natural length of this spring.
Graph and find the arclength along the curve described by the following parametric equations between the points where \( t = -1 \) and \( t = 2 \). Evaluate the integral on your calculator.

**Table 1:** \( x = 3t - t^3, \quad y = 3t^2 \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
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<tr>
<td>0</td>
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<td>1</td>
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<td>2</td>
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Find the arc length along the curve \( x = \sin(4t), \quad y = \cos(3t) \) between times \( t = \frac{\pi}{4} \) and \( t = \frac{2\pi}{3} \).
Derive the arclength formula for parametric equations in two dimensions and adapt that formula for curves where \( y = f(x) \).