Key Concept: **L’Hospital’s Rule**

**How do I apply the rules? What work do I need to show?**

Examples 1-3 show limits of form \(\frac{0}{0}\) or \(\pm \infty\).

Example 4 shows a limit of form \(\pm \infty \cdot 0\).

Example 5 shows a limit of form \(\infty - \infty\).

Examples 6 and 7 show limits of form \(1^\infty, 0^0, \text{ or } \infty^0\).

**Example 1**

Evaluate \(\lim_{x \to 0} \frac{\sin(2x)}{x}\).
Example 2

Evaluate $\lim_{t \to \infty} \frac{\tan \left( \frac{2}{t} \right)}{\ln \left( 1 + \frac{3}{2t} \right)}$ using L’Hôpital’s rule (or its corollary) where appropriate.
Example 3
Evaluate \( \lim_{x \to 0} \frac{2\sin(x) - 2x}{x^3} \) using L’Hopital’s rule (or its corollary) where appropriate.
Example 4

Evaluate \( \lim_{x \to \infty} \frac{2x}{\ln(3x + e^{5x})} \) using L’Hospital’s rule (or its corollary) where appropriate.
Example 5

Evaluate \( \lim_{x \to \infty} x \cdot \ln \left( \frac{x - 1}{x + 1} \right) \) using L’Hopital’s rule (or its corollary) where appropriate.
Example 6
Evaluate \( \lim_{\theta \to 1} \left( \frac{\theta}{\theta - 1} - \frac{1}{\ln(\theta)} \right) \) using L’Hopital’s rule (or its corollary) where appropriate.
Example 7
Evaluate \( \lim_{x \to 2} (x - 1)^{\tan(\pi/x)} \) using L’Hopital’s rule (or its corollary) where appropriate.
Example 8

Evaluate \( \lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^{bx} \) using L'Hopital's rule (or its corollary) where appropriate.
Example 9

Evaluate \( \lim_{x \to \infty} \frac{2x - \sin(x)}{7x + \cos(x)} \) using L’Hospital’s rule (or its corollary) where appropriate.
Horizontal Asymptotes

If \( \lim_{x \to \infty} f(x) = k \) or \( \lim_{x \to -\infty} f(x) = k \), where \( k \) is a real number, then the line \( y = k \) is a horizontal asymptote on the graph of \( y = f(x) \).

Example 10

A graph of the function \( y = x - \ln(x + 2e^x) \) is shown in Figure 3. What is the value of \( k \) in the equation of the horizontal asymptote shown on the graph?