MTH 252 Graded HW 2  

This assignment is due at 6:00 PM on Tuesday, January 20

You may work on this assignment with your classmates or anybody else you please. You may get help from a tutor or even the instructor. What you may not do is simply copy somebody else’s work—that completely obviates the purpose of the assignment. If you forget to complete the assignment before it is due, do not simply copy someone else’s paper and turn that in ... that is not “working together,” that is taking credit for somebody else's work.

You should take all derivatives and perform all non-trivial algebra on your calculator ... to earn full credit you must outline your work and show the steps in a manner consistent with that demonstrated and discussed during lecture. That includes, but is not limited to, completely simplifying all derivative formulas.

1. Find the stationary numbers of the function \( f(x) = x^5 - 7x^3 + 20x^4 - 30x^3 + 25x^2 - 11x + 2 \).

   Perform a second derivative test at each of those numbers and state appropriate conclusions.

\[
\begin{align*}
  f'(x) &= (x-1)^4 (6x - 11) \\
  f'(x) &= 0 \text{ at } 1 \text{ and } \frac{11}{6}, \text{ so the stationary numbers of } f \text{ are } 1 \text{ and } \frac{11}{6}. \\
  f''(1) &= 0 \text{ and } f''(\frac{11}{6}) = \frac{625}{16}
\end{align*}
\]

\[
\begin{align*}
  \text{Since } f'(\frac{11}{6}) &= 0 \text{ and } f''(\frac{11}{6}) > 0, \text{ we know that } f \text{ has a local minimum point at } \frac{11}{6}. \\
  \text{Since } f''(1) &= 0, \text{ the second derivative test is inconclusive at } 1.
\end{align*}
\]
2. Use appropriate calculus-based techniques to find the absolute maximum value of the function 
\[ k(x) = x^{\frac{1}{3}}(x - 7)^2 \] over the interval [5, 9]. Round values to the nearest 100th (where rounding is appropriate).

\[ \frac{5x - 21}{3(x - 7)^{\frac{1}{3}}} \]

Over [5, 9], \( k'(x) \) never equals zero and \( k'(x) \) is undefined at 7.

\[ \therefore \text{The only critical number of } k \text{ over [5, 9] is 7.} \]

**Table 2:** \( k(x) = x(x - 7)^{\frac{1}{3}} \)

<table>
<thead>
<tr>
<th>x</th>
<th>k(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3\sqrt{4}</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>9\sqrt{3}</td>
</tr>
</tbody>
</table>

\[ \therefore \text{As seen in Table 2, over [5, 9] the absolute maximum value of } k(x) \text{ is } 9\sqrt{3} \text{ (about 14.84).} \]