You may hold onto this portion of the test and work on it some more after you have completed the no calculator portion of the test.

On this portion of the test you are expected to take all derivatives and perform all nontrivial algebra and arithmetic using your calculator.

Don’t forget that all derivative formulas need to be completely simplified; this includes, but is not limited to, finding the completely factored form of the derivative formula.

To earn full credit, your solutions need to include all relevant information and exclude any irrelevant information. You should use appropriate calculus based techniques as illustrated in class.

1. Consider the function $f(x) = e^{\sqrt{x-2}} \sqrt{x-2}$. Find the critical numbers of $f$, showing all of the details that go into your determination. Make sure that you present your work in a manner that is consistent with that demonstrated and discussed in class. (10 points)
2. Find the inflection points on the function \( f'(x) = (x - 1)^3 (x + 7)^6 \). Make sure that you present your work in a manner that is consistent with that demonstrated and discussed in class. (10 points)
3. Find the stationary numbers of the function \( f(x) = x\sqrt{9 - x} \). Then perform a second derivative test at each stationary number and state the appropriate conclusions. Make sure that you present your work in a manner that is consistent with that demonstrated and discussed in class. (8 points)
4. Suppose that a fish swims upstream (against the current) at the speed of \( x \) relative to the water and that the current speed of the river is \( v \). It can be shown that the amount of energy required for the fish to swim a distance, \( d \), up the river is given by the function \( E(x) = \frac{k d x^3}{x - v} \) where \( k \) is a proportionality constant whose value depends upon properties of the fish and the units used while working the problem.

Determine whether or not there is a speed at which the fish can swim that minimizes the amount of energy required to swim a distance \( d \). If there is an energy minimizing speed, state the value of that speed (in terms of the constants in the energy equation). Please note that \( v, d, \) and \( k \) are all **positive** constants. Make sure that you show all relevant work and make sure that your conclusion is clear. (10 points)
3. Find the absolute maximum value of the function \( f(x) = 2x^3 - 3x^2 - 12x + 2 \) over the interval \([0,3]\). Make sure that you show work consistent with that illustrated and discussed during class and make sure that your conclusion is clear and on point. (10 points)
4. Formally establish each limit. To earn full credit you need to notate and organize your work in a manner consistent with that illustrated and discussed in class. Don’t forget to state the form of the limit before beginning the problem and don’t forget to state the form of the limit before each and every execution of L’Hopital’s Rule. This problem continues on page 4. (18 points total)

\[ \lim_{{x \to 0}} \frac{e^{5x} - 1 - 5x}{4x^2} \]
b. Evaluate $\lim_{{x \to \frac{\pi}{2}}} x^{\csc(x)}$.

**Reminder:** $\csc(t) = \frac{1}{\sin(t)}$
5. For each of the following statements, circle T if the statement is true and circle F if the statement is false. In questions where I refer to “the first step” I mean that you can perform the operation without doing any intervening algebra what-so-ever. (6 points)

a. T or F \( \lim_{{x \to 4}} \frac{x - 3}{x - 4} \) has indeterminate form.

b. T or F \( \lim_{{x \to \infty}} \left( \frac{1}{x} \right)^{\sin(x)} \) has indeterminate form.

c. T or F \( \lim_{{x \to 0^+}} \left( \frac{1}{x} \right)^{\sin(x)} \) has indeterminate form.

d. T or F You can legitimately perform L’Hopital’s Rule (or corollary to L’Hopital’s rule) as your first step when evaluating \( \lim_{{\theta \to 0}} \frac{\sin(\theta)}{\sin(\theta)} \).

e. T or F You can legitimately perform L’Hopital’s Rule (or corollary to L’Hopital’s rule) as your first step when evaluating \( \lim_{{x \to 0^+}} \left[ xe^{1/x} \right] \).

f. T or F You can legitimately perform L’Hopital’s Rule (or corollary to L’Hopital’s rule) as your first step when evaluating \( \lim_{{t \to e}} \frac{\ln(t)}{t - e} \).
2. Evaluate, without the use of a calculator, \[ \int_0^\infty \frac{1}{e^{-4x} + e^{4x}} \, dx \]. After making the substitution \( u = e^{4x} \), show all relevant work. (12 points)
3. Evaluate, without the use of a calculator, \( \int_{0}^{4} \frac{dx}{(5x - 2)^2} \). Show all relevant work. (12 points)