This test is due at 6:00 pm on **Wednesday**, April 24, 2013

You may not receive any help of any kind on any question on this test, nor may you receive any help of any kind on any question similar to a question on this test — get your practice homework done before you even look at the questions on this test. Please note that copying somebody else’s answer falls into the category of getting help with a question. Please note that trolling the internet for solutions constitutes cheating. Please note that failure to adhere to these rules will result, among other things, in your having to write all subsequent test answers in Haiku.

You may ask Mr. Simonds if you do not understand what a question is asking. Please let him know as well if you think that the test has a substantive typo. I will post any necessary fixes on my web site.

To earn full credit on any given problem your work must be presented in a manner consistent with that demonstrated and discussed during both lecture and lab. Remember to show your algebraic steps one step at a time.

1. The three “sub-properties” that must be true about the function $f$ at the number $a$ if $f$ is continuous at $a$ are:
   i. $f$ must be defined at $a$
   ii. $\lim_{x \to a} f(x)$ must exist
   iii. $\lim_{x \to a} f(x)$ must equal $f(a)$

   State each value of $x$ where the function $f$ shown in Figure 1 is discontinuous. For each discontinuity, state (by number) each of the sub-properties of continuity that fail at that value of $x$. (10 points)

   ![Figure 1: $f(x)$](image)

2. Referring to the function in Figure 1, fill the appropriate number or symbol into each blank and then indicate whether or not the limit exists. No other work need be shown. (3 points each)

   a. $\lim_{x \to \infty} f(x) = \underline{1}$
      
      Does the limit exist? **Yes**

   b. $\lim_{x \to 4} (2f(x) + 4) = \underline{8}$
      
      Does the limit exist? **Yes**

   c. $\lim_{x \to 8} f(x) = \underline{1}$
      
      Does the limit exist? **Yes**
3. Use limit laws to formally establish the value of \( \lim_{t \to a} \sqrt[4]{\frac{16e^{4t}}{25e^{-6t} + 9e^{4t}}} \). (14 points)

To earn full credit your work must include all relevant limit law steps accompanied by a statement of the limit law number(s) used in each of those steps (as shown in class).

\[
\begin{align*}
\lim_{t \to a} \sqrt[4]{\frac{16e^{4t}}{25e^{-6t} + 9e^{4t}}} &= \lim_{t \to a} \sqrt{\frac{16e^{4t}}{25e^{-6t} + 9e^{4t}}} \cdot \frac{e^{-4t}}{e^{-4t}} \\
&= \lim_{t \to a} \sqrt{\frac{16}{25e^{-6t} + 9}} \\
&= \sqrt{\lim_{t \to a} \frac{16}{25e^{-6t} + 9}} \\
&= \sqrt{\frac{16}{25 \cdot e^{-6a} + 9}} \\
&= \sqrt{\frac{16}{0.0 + 9}} \\
&= \sqrt{\frac{16}{9}} \\
&= \frac{4}{3}
\end{align*}
\]
4. Use limit laws to formally establish the value of \( \lim_{x \to \frac{\pi}{2}} \frac{\sin(2x) + \cos(2x) + 1}{\cos(x)} \). (14 points)

To earn full credit your work must include all relevant limit law steps accompanied by a statement of the limit law number(s) used in each of those steps (as shown in class).

\[
\lim_{x \to \frac{\pi}{2}} \frac{\sin(2x) + \cos(2x) + 1}{\cos(x)} = \lim_{x \to \frac{\pi}{2}} \frac{2\sin(x) \cos(x) + 2\cos^2(x) - 1 + 1}{\cos(x)} \\
= \lim_{x \to \frac{\pi}{2}} \frac{2\sin(x) \cos(x) + 2\cos^2(x)}{\cos(x)} \\
= \lim_{x \to \frac{\pi}{2}} \frac{2\cos(x)(\sin(x) + \cos(x))}{\cos(x)} \\
= \lim_{x \to \frac{\pi}{2}} [2(\sin(x) + \cos(x))] \quad \text{LLA7} \\
= \lim_{x \to \frac{\pi}{2}} (\sin(x) + \cos(x)) \quad \text{LLA3} \\
= 2 \left( \lim_{x \to \frac{\pi}{2}} \sin(x) + \lim_{x \to \frac{\pi}{2}} \cos(x) \right) \quad \text{LLA1} \\
= 2 \left( \sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right) \right) \quad \text{LLA10} \\
= 2 \left[ \sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right) \right] \quad \text{LLAR1} \\
= 2
5. a. Find \( \lim_{{h \to 0}} \frac{\sqrt{25 + h} - 5}{h} \). You need to show all of the relevant steps up through the application of Limit Law 7. You do not need to formally go through the remaining limit law steps – you may simply replace \( h \) with zero once you get to that point in the process. (8 points)

\[
\lim_{{h \to 0}} \frac{\sqrt{25 + h} - 5}{h} = \lim_{{h \to 0}} \left[ \frac{\sqrt{25 + h} - 5}{h} \cdot \frac{\sqrt{25 + h} + 5}{\sqrt{25 + h} + 5} \right] \\
= \lim_{{h \to 0}} \frac{25 + h - 25}{h(\sqrt{25 + h} + 5)} \\
= \lim_{{h \to 0}} \frac{h}{h(\sqrt{25 + h} + 5)} \\
= \lim_{{h \to 0}} \frac{1}{\sqrt{25 + h} + 5} \\
= \frac{1}{\sqrt{25 + 0} + 5} \\
= \frac{1}{10}
\]

b. A graph of the function \( y = \sqrt{x} \) is shown in Figure 2. The value you found in part (a) of this question tells you something relevant about this graph. Carefully illustrate what it is that the value found in part (a) tells you about the graph. Use words and drawings as necessary. (3 points)

The slope of the tangent line to the curve \( y = \sqrt{x} \) at the point where \( x = 25 \) is \( \frac{1}{10} \).

Figure 2: \( y = \sqrt{x} \)
6. Find and simplify the difference quotient for \( k(x) = \frac{10}{6-x} \) showing each step in the simplification process. (12 points)

The difference quotient for \( k(x) = \frac{10}{6-x} \):

\[
\frac{k(x+h) - k(x)}{h} = \frac{\frac{10}{6-(x+h)} - \frac{10}{6-x}}{h}
\]

\[
= \frac{\frac{10}{6-x-h} \cdot \frac{6-x}{6-x} - \frac{10}{6-x} \cdot \frac{6-x-h}{6-x}}{h}
\]

\[
= \frac{60 - 10x - 60 + 10h}{(6-x-h)(6-x)} \cdot \frac{1}{h}
\]

\[
= \frac{10h}{(6-x-h)(6-x)} \cdot \frac{1}{h}
\]

\[
= \frac{10}{(6-x-h)(6-x)} \text{ for } h \neq 0
\]
7. The Great Lakes are believed to experience small tides. Let’s suppose that on one particular day the high tide for Lake Erie in Sandusky, OH occurred at 1:00 PM and that for the next six hours the deviation from the low tide water mark was given by the function \( w(t) = \frac{4}{t+1} \) where \( w(t) \) is measure in centimeters and \( t \) is measured in hours. The difference quotient for \( w \) simplifies as \( \frac{w(t+h) - w(t)}{h} = -\frac{4}{t(t+h)} ; h = 0 \); you do not need to establish that – it’s been done for you.

Use the difference quotient for \( w \) to determine the average rate of change in \( w \) over the time interval \([3.5, 5]\); round the value to the nearest 100th. Then state the practical meaning of the value (including unit). To earn full credit you need to state your times using “normal language.” The rest of your answer needs to be correct as well. (10 points)

\[
(x = 3.5, y = 1.5) \quad \text{and} \quad (x = 5.5)
\]

\[
\frac{w(t+h) - w(t)}{h} = -\frac{4}{t(t+h)}
\]

\[
= -\frac{4}{2.5(5)}
\]

On that day, the Lake Erie tide at Sandusky, OH, recorded at an average rate of .23 cm/hr between 4:30 PM and 6:00 PM.

8. Draw onto Figure 3 a function, \( f \), that satisfies each and every one of the following properties. Make sure that you draw and appropriately label all asymptotes. (14 points)

- \( f \) does not have more than three discontinuities
- The only x-intercepts occur at \(-4\) and \(2\)
- The y-intercept is \((0, 3)\).
- \( f(-5) = 2 \)
- \( \lim_{x \to 0^+} f(x) = -3, \lim_{x \to 4} f(x) = -\infty, \lim_{x \to \infty} f(x) = 0 \)
- \( f \) has constant slope on \((-\infty, -2)\) and then again on \([-2, 0]\).

![Figure 3: f](image-url)
9. Consider the function \( f(x) = \begin{cases} \frac{5}{x-10} & \text{if } x < 5 \\ \frac{5}{5x-30} & \text{if } 5 < x < 7 \\ \frac{x-2}{x-12} & \text{if } x \geq 7 \end{cases} \). State the values of \( x \) where each of the following occur. If a stated property doesn't occur, make sure that you state that (as opposed to simply leaving the answer space blank). No explanation necessary. (1.5 points each)

a. At what values of \( x \) is \( f \) discontinuous?

\[ 5, 6, \text{ and } 7 \]

b. At what values of \( x \) is \( f \) continuous from the left but not the right?

There are no such values.

c. At what values of \( x \) is \( f \) continuous from the right but not the left?

7

d. At what values of \( x \) does \( f \) have removable discontinuities?

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